Lost in Transition: Financial Barriers to Green Growth*

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Abstract

Green innovation offers a solution to climate change without compromising living standards. Yet the share of climate-enhancing innovations in total patents, after booming for two decades, has seized to grow since the Global Financial Crisis. We develop a quantitative framework in which firms direct innovation towards green or polluting technologies, and become better at innovating in technologies that they have previously succeeded in. This causes mature, incumbent firms to predominantly innovate in polluting technologies. When green technologies become more attractive, e.g. due to a carbon tax, young firms are responsible for a large share of the transition to green innovation. As young firms are financially constrained, a credit shock harms their innovation, bringing the green transition to a halt. We validate the theory with two empirical exercises. First, we use micro data to provide causal evidence that tight credit disproportionately affects green innovation, through its effect on young firms. Second, we show that contractionary monetary policy shocks have a significantly larger effect on green patenting than non-green patenting, in line with the model. Quantifying the model, we find that tight credit can explain around 60% of the recent slowdown in the rise of green patenting. This translates to a cumulative increase in emissions by half a year of the initial (high pollution) steady state.

Keywords: Climate Change, Productivity, Endogenous Growth, Innovation, Creative Destruction.

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1 Introduction

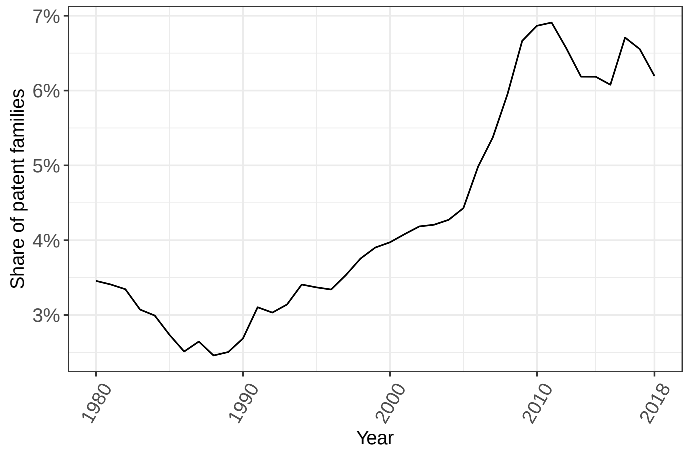
Climate change is widely acknowledged as one of the most significant challenges facing humanity in the 21st century. The overwhelming scientific evidence points to human activities, particularly the burning of fossil fuels, as the primary cause of the unprecedented warming of the Earth's atmosphere, oceans, and land (IPCC 2021). To fight climate change some have argued for negative growth, based on the historical fact that temperature and aggregate CO2 emissions worldwide started to increase precisely at the time of the growth takeoff in the nineteenth century. This, however, would require dramatic declines in consumption and living standards. A more promising route, one that reconciles climate with the quest for prosperity, is green innovation: to discover cleaner sources of energy, cleaner products and cleaner production technologies. Yet the share of green patents in total patents, which rose rapidly during the 1990s and 2000s, has seized growing in the aftermath of the Global Financial Crisis (Figure 1).

This paper shows that an innovation-lead transition to green innovation is sensitive to financial disruptions. We posit *path dependence* in the direction of firms' innovation decisions as the driver of this sensitivity. Path dependence refers to the fact that firms build up expertise in directions of innovations that they have previously successfully innovated in. As pointed out by Acemoglu et al. (2012), such path dependence implies that incumbents have a comparative advantage to innovate in technologies that rely on fossil fuels for production. Firms that innovated in these polluting (or "dirty") technologies in the past will thus tend to continue innovating in dirty technologies in the future, taking advantage of their accumulated knowledge, even when consumer preferences or policy interventions reduce the profitability of dirty production. Young firms and entrants, on the contrary, have a comparative advantage in green innovation. As a result, green innovation at the start of a transition concentrates in young firms.

The fragility of green innovation to tight credit then arises from the fact that these young firms are particularly financially constrained. Young firms have yet to establish a track record and transaction history, raising monitoring costs and reducing access to finance from banks (Gertler and Gilchrist 1994). On top of that, they typically have no access to bond and equity markets, preventing them from substituting bank loans with other sources of funds. As a result, a tightening of credit constraints from an increase in (policy) interest rates or an overall reduction in credit supply particularly harms their ability to invest. The interaction of young firm's financial constraints and their endogenous comparative advantage in green innovation drives the overall detrimental effect of financial tightness on the green transition.

We proceed in two parts. In the first part of the paper, we develop a model of innovation-led growth with creative destruction and firm dynamics in the spirit of Klette and Kortum (2004). We depart from the standard models in two main respects. First, firms separately decide how much to invest in green and dirty innovation when expanding their activities to new product lines. The direction of innovation determines whether their new goods are produced with a polluting (dirty) or a non-polluting (clean) technology. This innovation process exhibits path-dependence: the higher the number of lines in which a firm currently produces using a green (respectively a dirty) technology, the lower the cost of further green (respectively dirty) innovation. As a result, total green innovation is a function of both the profitability of producing with green technologies and the fraction of the economy's products that is already produced

Figure 1: Global Trend: Green Patents as a Percentage of Total Patents



Notes: % patents in environmental management, water-related adaption technologies, biodiversity and ecosystem health, climate-change mitigation (OECD classification - Hascic and Migotto 2015) - PATSTAT data. Appendix A Figure A1 shows that there is a similar break in green patenting trends from 2011 when only including triadic patents – that is, patents filed at each of the European Patent Office, the U.S. Patent Office and the Japan Patent Office – as well as when weighing patents by forward citations, when only including OECD countries, or when considering trends within individual countries.

green. Second, firms in our model face a borrowing cost that is driven by ex-post moral hazard and costly monitoring. This monitoring cost declines as firms age, because the firm's track records enable creditors to learn how to monitor a firm more efficiently. This makes younger firms' innovation more elastic to overall credit conditions than mature firms.

In the second part, we validate the model with causal evidence on the sensitivity of green innovation to financial constraints. We establish three facts. First, using data on the banks from which firms initially borrow, as in (e.g.) Chodorow-Reich (2014) and Huber (2018), we find that firms exposed to a tightening of credit during the Global Financial Crisis see a decline in their patenting *solely* in green technologies. Other technologies are not significantly affected. The decline emerges from 2014 onwards, coinciding with an overall decline in green innovation in Figure 1. Second, we show that the negative effect of financial tightness on green innovation is exclusive to younger firms - older firm's green innovation rates are unaffected. Third, we show that younger firms were both responsible for a large share of green innovation *prior* to the crisis (but a small share afterwards) and were generally more inclined to innovate green.

We then quantify the model to understand the effects of financial tightness on the green transition. Our analysis focuses on a transition where the economy moves from a steady state in which most products are produced with dirty, polluting technologies, to one where most products are produced with clean technologies. The driver of the transition is policy: an increase in the relative profitability of producing goods with a clean rather than a dirty technology, for instance through the introduction of a carbon tax.[1]

[1] The existing literature on climate and green innovation (e.g., Popp 2002; Acemoglu et al. 2012) has mostly emphasized the role of direct state intervention: carbon taxes and tariffs, subsidies to green innovation, competition policy and industrial policy. Aghion et al. (2023a) and Hart and Zingales (2017) look at the role of civil society in inducing firms to innovate green. Aghion et al. (2023a) analyzes the role of consumers' demand for better environment, whereas Hart and Zingales (2017)'s focus is on shareholders' concern for corporate social responsibility. Criscuolo et al. (2023) provide an elaborate discussion on potential industrial strategies to achieve net zero.

We structurally estimate the model's initial high-pollution steady state to match the patterns of innovation between 1985 and 1995 and then implement a pollution tax that is sufficiently large to reduce emissions by 65%. As the vast majority of patents in the initial steady state is in the dirty technology, incumbents initially have limited incentives alter the direction of innovation towards green innovation. As a result, the model matches the empirical fact that young firms initially invest in green innovation at comparatively high rates, and that most green innovations are produced by young firms.

We then simulate a financial crisis that occurs early in the green transition. The financial crisis primarily reduces the ability of young firms to fund innovation, thus forcing them cut their research and development expenditures. Because of the direction of young firms' innovation, the model predicts a much larger aggregate elasticity of green patents with respect to financial tightening, despite the fact that the financial crisis does not directly affect green projects. We show that a financial crisis is able to explain a significant share of the decline in green innovation. In our main analysis, where we model the decline in credit conservatively, we are able to explain around 60% of the slowdown in green patenting after 2008. The financial crisis causes a persistent increase in emissions, causing the stock of global emissions to be the equivalent of more than half a year of pre-transition emissions higher as a result of the crisis.

The exercise yields two broader theoretical insights. First, we show that the effect of policies and shocks are different when the economy is transitioning to a new steady state. Most analyses of policy and welfare in endogenous growth models focus on effects along the balanced growth path, under the premise that key mechanisms still apply.[2] We show that this may not be the case. In our model, a financial shock has equal effects on green and dirty innovation in the balanced growth path, as the age distribution (and thus exposure to credit shocks) is similar for incumbents with predominantly green and dirty products in their portfolio. Yet along a transition to a lower-pollution steady state, green innovation is endogenously more sensitive to financial shocks, suggesting policy intervention is appropriate. Second, we show that credit disruptions are particularly costly during a transition. Whenever the composition of innovation is changing direction to a technology that is characterized by path dependence, a tightening of credit constraints slows the transition due to its effect on young (transition-driving) firms.

When quantifying the effect of the financial crisis on the green transition, we deliberately do not target the overall decline in green patenting. This is because other changes in policy in the aftermath of the crisis (Criscuolo et al. 2023), as well as declines in fossil fuel and tradable emission permit prices (Acemoglu et al. 2023), have likely also contributed to the decline in green innovation. Given the disproportionate decline in green innovation among young firms, as well as our micro-econometric evidence, we conclude that tight credit is a significant, but complementary, driver of the trend in Figure 1.

Related literature This paper relates to several strands of literature. Our theoretical contribution is closest to the literature that jointly studies endogenous technological change and the transition to clean production. The seminal reference is Acemoglu et al. (2016), from which we take the production side of the economy. Our key novelty is that we introduce a path dependence that ties the economy's capacity

[2]Notable exceptions include Acemoglu et al. (2016), Aghion et al. (2023b), and De Ridder (2024).

to engage in green innovation to the prevalence of green production. This causes green innovation to concentrate among young firms in the transition, which in turn drives its sensitivity to tight credit.

More broadly, we contribute to the literature that analyzes the interaction between climate and directed technological change after Acemoglu et al. (2012).[3] Existing work in this literature primarily focuses on how policy can induce an optimal transition to clean production (see, e.g., Hémous 2016, Aghion et al. 2016, Fried 2018, Acemoglu et al. 2023). Hémous and Olsen (2021) provide a recent review. We contribute to this literature by introducing credit constraints and firm dynamics, looking at how credit tightening can slow down the transition of the overall economy towards a clean economy by directly affecting green innovation by younger and smaller firms.

Our theoretical exercise builds on models where financial frictions impede optimal innovation decisions (e.g. King and Levine 1993, Rajan and Zinglas 1998, Aghion et al. 2018, and more recently Akcigit et al. 2023). A related literature particularly focuses on financial crises. Garcia-Macia (2015) claims that firms are unable to fund investment in intangible assets during financial crises, as their collateral value is hard to collect. Similarly, the models in Ates and Saffie (2013, 2014) claim that financial turmoil affects technological progress through the ability of banks to observe project quality under imperfect information. In Queraltó (2013), financial crises increase the costs of financial intermediation through balance sheet deterioration á la Gertler and Kiyotaki (2010), which reduces the entrance of entrepreneurs. Schmitz (2021) adds that a crisis' effect on innovation is amplified by the fact that small and young firms are most affected by credit tightness, as these produce more radical innovation. We contribute to this literature by looking at the effect of credit constraints on the direction – green versus dirty – of innovation, in younger versus mature firms, and from there at the overall effects of credit tightening on the speed of energy transition toward a low-pollution steady state.

The key assumption in our model that young firms face greater financial constraints is also rooted in a long literature. Gertler and Gilchrist (1994) note that informational frictions in finance are particularly strong for young firms. This is because young firms have yet to establish a sufficiently long credit history, as pointed out by (e.g.) Evans and Jovanovic (1989), Haltiwanger et al. (2013), Davis and Haltiwanger (2019) and Chaney et al. (2012).[4] Recent evidence supports this idea. Cloyne et al. (2023) find that young firms' investments are more responsive to monetary policy shocks, a result confirmed for employment by Bahaj et al. (2019). Clymo and Rozsypal (2022) similarly find that young firms exhibit more business cycle sensitivity, consistent with tighter financial constraints.

We consider our theoretical exercise complementary to papers that explain why clean technology adoption might be difficult for financially constraint firms. Lanteri and Rampini (2023), for example, show that older shipping firms were more likely to adopt clean technologies in 2022 than younger firms,

[3]This work, in turn, builds on the long literature on growth and climate change, starting with Nordhaus (1994). See Golosov et al. (2014) and the reviews by Caggese et al. (2023) and Hassler et al. (2018) for more recent developments.

[4]Firm size plays no direct role in a firm's financial constraints in our model. This is because, once conditioning on the fact that large firms are on average older, we find no significant difference of the effect of the Global Financial Crisis on green innovation between small and large firms. This finding is consistent with evidence in Ferreira et al. (2023), who show that financially constrained firms are present across the firm-size distribution. Siemer (2019) similarly points out that the large effect of the Global Financial Crisis on small U.S. firms is driven by the fact that small firms are on average younger.

and rationalize this with a model in which green adoption comes at high up-front costs. Our contribution is that even in absence of *any* innate difference in the cost of financing green or dirty innovation, green innovation is endogenously more sensitive to financial constraints when the economy is in transition.

Our empirical analysis relates to the recent literature on the financial costs of climate change and on green finance. Bolton et al. (2022) show that financiers require compensation for climate risk. Our focus is instead on how finance – or the lack of it – affects firms' choice to innovate green versus dirty, and from there the steady state firm-size distribution and aggregate growth and welfare. We also relate to De Haas et al. (2024), who show that firms that were rejected from a loan application were less likely to adopt green management technologies and to invest in green practices. De Haas and Popov (2020) find that industries in countries with better-developed financial systems transition faster to the use of low-carbon production technologies. Our contribution over and above these papers is to demonstrate that green innovations are particularly severely hampered by financial frictions.

A related literature notes that there has recently been a dramatic rise in "green investors": investors that allocate a large fraction of their capital to climate-change related projects. Bolton and Kacperczyk (2021) show that low-emission firms have lower expected returns on U.S. capital markets, implying that their valuations are higher and cost of capital are lower. Berk and van Binsbergen (2021) finds that the impact of green investors on cost of capital remains quantitatively small. Gormsen et al. (2021) argue that the green wedge in capital costs has increased rapidly between 2016 and 2022. Accetturo et al. (2022) show that green investments are responsive to changes in credit supply, consistent with our findings. Patozi (2023) notes that the rise of green investors might affect the transmission of monetary policy. A full review on the interaction between climate change and finance is provided in Giglio et al. (2021).

Our empirical strategy builds on the literature that identifies the real effects of financial tightness by exploiting variation in the lenders that firms rely on. Examples include Chodorow-Reich (2014), Acharya et al. (2015), Bentolila et al. (2015), Franklin et al. (2015) and Giroud and Mueller (2015), who analyze the employment effects of tightening of credit supply using variation in firm-level shock-exposure. It is similarly related to Almeida et al. (2012), Greenstone et al. (2014), Adelino et al. (2015), Aghion et al. (2015), Paravisini et al. (2015), and De Ridder (2019). These papers use exposure to credit shocks to analyze the effect on employment, investments, exports, output and patenting. To our knowledge, ours is the first paper to that methodology to study the differential effect of tight credit on (non) green patenting.

Outline The remainder of the paper is organized as follows. Section 2 develops our model of creative destruction and firm dynamics with credit constraints and green versus dirty innovation. Section 3 provides evidence of an age- dependent effect of financial tightening on green versus dirty innovation, by using differential impact of the Global Financial Crisis across firms as a natural experiment. Section 4 shows that green innovation is also more sensitive to monetary policy shocks than non-green innovation. Section 5 presents the quantification while Section 6 uses the model to evaluate the impact of credit tightening on firms' decision to innovate green versus dirty, on the resulting steady-state firm size distribution, and on aggregate growth and welfare. Section 7 concludes.

2 A model of financial constraints, green innovation, and firm dynamics

This section develops a framework to explain the relationships between financial conditions, green innovation, and firm dynamics. At the heart of our theory is the idea that finance has a *state-dependent* effect on green innovation: only during a transition to a low-emission economy will a tightening of financial conditions disproportionately harm green innovation. To arrive at this conclusion, we device a Schumpeterian model with directed technological change. The model builds on Klette and Kortum (2004) in the sense that firms produce one or more products, and successful innovation enables firms to expand their product portfolio. Firms innovate by directing research and development (R&D) expenses to either clean technologies or dirty technology, where directed innovation determines the kind of inputs that firms are required to use in the production of a good upon successful innovation.

Our framework relies on two key assumptions. First, we assume that innovation comes with path dependence – firms are more likely to direct innovation towards clean (or dirty) technology if they have accumulated a large stock of past successful clean (or dirty) innovations. Second, firms' innovation is subject to a financial cost, which decreases in age. The first assumption yields that at the beginning of a green transition, young firms have a comparative advantage in green innovation and are thus responsible for a disproportionate fraction of clean growth. The second assumption then yields that a tightening of credit particularly hampers the transition, as young firms cut their investments in green innovation.

2.1 Preferences and production

Time is continuous and there is a mass L of infinitely-lived households with identical preferences. The representative household has intertemporal utility function

$$U = \int_0^\infty e^{-\rho t} \ln C_t \, dt,$$

where ρ denotes the discount rate and Y_t measures aggregate output at time t, the subscript of which we omit when convenient. The log-utility assumption leads to the standard Euler equation in steady-state growth: $r = \rho + g$, where r denotes the interest rate and g is the steady-state growth rate. There is no disutility of labor, such that the household supplies its full time endowment to firms.

The household's instantaneous utility depends on consumption aggregate C_t, which is a competitively aggregated basket of a continuum of final goods Y_{jt} with $j \in [0,1]$, according to:

$$\ln C_t = \int_0^1 \ln Y_{jt} \, dj.$$

Each good j can be produced by the set of firms that owns the production technology, a patent, to produce that good. A patent enables a firm to produce good j with either a clean (c) or dirty (d) production technology, and at some level of quality q_{ij}^h, $h \in (c,d)$. Dirty and clean goods are perfect substitutes in the eyes of consumers, such that each final good's output can be produced with either input along:

$$Y_{jt} = Y_{jt}^c + Y_{jt}^d,$$

where Y_{jt}^c is produced using clean intermediate inputs and Y_{jt}^d is produced using dirty intermediate inputs. Ignoring time subscripts for notational convenience, we posit that:

$$Y_j^h = \sum_{i \in I_j^h} q_{ij}^h y_{ij}^h,$$

where I_j^h is the set of firms that owns a patent of type h to produce good j, while y_{ij}^h is firm i's output of j.

Clean and dirty products differ in their production function. A firm that produces a good using a clean technology only requires production labor l_{ij}^c to do so, while production with a dirty technology relies on both labor l_{ij}^d and a fossil input o_{ij}. This follows the demarcation of clean and dirty production in Acemoglu et al. (2016). The clean and dirty production functions respectively read as:

$$y_{ij}^c = l_{ij}^c \quad \text{,and} \quad y_{ij}^d = \left(l_{ij}^d\right)^\eta \cdot \left(o_{ij}\right)^{1-\eta},$$

where $\eta \in (0,1)$ governs the factor shares in production with the dirty technology.

As firms choose production labor and fossil inputs without intertemporal considerations or adjustment costs, a firm's optimal use of fossil inputs in dirty production is given by

$$o_{ij} = \left(\frac{1-\eta}{\eta}\frac{w}{p^0}\right) l_{ij}^d, \tag{1}$$

where p^0 is the relative price of the dirty input in units of numeraire, which we take to be equal to the marginal cost of extraction of the dirty input. Thus an increase in p^0 reflects an increase in the cost of using the dirty input, for example as a resulting from an increased emission tax.

Equation (1) tells us that firms' use of dirty inputs increases in their output elasticity and falls in the relative price of the dirty input. Inserting the first-order condition into the production function gives

$$y_{ij}^d = l_{ij}^d / \tilde{p}^0,$$

where the relative cost of producing with a dirty input is given by

$$\tilde{p}^0 = \left(\frac{\eta}{1-\eta}\frac{p^0}{w}\right)^{1-\eta}.$$

In subsequent sections we raise p^0, and thereby in $\tilde{p}^0$, to initiate the transition towards a green economy. It follows from the above that the marginal cost mc^h of producing a input of type $h \in \{c,d\}$ is equal to

$$mc^h = \begin{cases} w & \text{if } h = c \\ w \times \tilde{p}^o & \text{if } h = d \end{cases}$$

2.2 Innovation and productivity growth

Firms expand their portfolio of green and dirty products by directing innovative resources towards green and dirty technologies. A firm that successfully innovates expands its product portfolio with a patent that enables the firm to produce a random good in the economy at a higher level of quality than its current producer, enabling the firm to become the new market leader. This process of quality improvement through vertical innovation is the driver of aggregate productivity growth.

2.2.1 Incumbent innovation

Innovation is directed in the sense that firms separately invest in green and dirty projects. In both of these technologies, the number of researchers rd_i^h that the incumbent firm deploys is related to the Poisson arrival rate of innovations X_i^h along the following function:

$$X_i^h = \left(\phi^h rd_i^h\right)^{\frac{1}{\psi}} \left(n_i^h\right)^{1-\frac{1}{\psi}},$$

where $\psi > 1$ determines the cost elasticity of R&D, $\phi^h > 0$ is a scalar, while n_i^h is the firm already produce of that particular technology – thus, a firm that produces a significant number of clean products is able to develop new clean patents more efficiently than a firm that is specialized in dirty production.

The fact that a firm's R&D efficiency in direction h rises with the number of goods it already produces in that direction reflects path-dependence in the choice between green and dirty innovation, and is our first key assumption. It is supported by existing empirical studies that find such path dependence. For example, Aghion et al. (2016) show that firms with a higher stock of dirty (respectively, green) past innovations are more likely to continue innovating in dirty (respectively, green) technologies in the future, using data from the automotive sector. More recently, Dugoua and Gerarden (2023) demonstrate that inventors with a track record of dirty innovation are unlikely to switch to producing green patents.[5]

The idea that a firm's R&D productivity depends on its current stock of products builds on Klette and Kortum (2004), who assume that firms become more efficient at innovation as they build organizational capital through their size. Practically, the assumption assures that firms grow at a constant expected rate regardless of size, in line with Gibrat's Law. We deviate from the traditional model by assuming that the positive effect of size on R&D productivity is technology-specific. That is, a firm's R&D productivity in either dirty or clean innovation depends on how many dirty or clean goods, respectively, it produces.[6]

A firm that successfully innovates obtains a template for a subsequent product innovation. The technology type h of the firm's product innovation is usually the technology in which the innovator directed its innovation. With a probability $1 - \alpha < 0.5$ the invention is of the alternative technology. This assures

[5] Dechezleprêtre and Hémous (2023) survey the literature on climate change and path dependence in innovation.

[6] Our setup also deviates from Acemoglu et al. (2016). In their model, dirty and green R&D productivity depends on the number of goods for which the firm owns the state-of-the-art dirty or green patents (respectively), irrespective of whether the good is actually produced with that product. This distinction is important: in our model, firms only benefit from higher R&D productivity for products that are actually produced, tying industrial composition to innovation capacity.

that firms without a track record in either dirty or green innovation will occasionally develop a patent of that type, and can be thought off as reflecting a cross-technology spillover.[7]

The template turns into an actual product innovation at the exogenous flow rate ζ. When this happens the innovating firm takes over production of a random product j by raising the quality at which it produces the good by a multiplicative step λ, which is assumed to exceed the level and inverse of $\tilde{p}^o$. Defining $q_{\bar{i}j}$ as the incumbent's quality, the innovator therefore produces at quality $q^h_{ij} = \lambda q_{\bar{i}j}$. That product innovations require time to materialize, is in line with the empirical evidence which shows a lag between R&D investments and the time when firms earn higher revenue from patenting.

2.2.2 Entrant innovation

There is a continuum of potential entrants that employ rd^{ec} researchers to develop green technologies and rd^{ed} to develop dirty technologies. The entry rates as a function of entrant R&D is given by

$$e^c = \left(\phi^c_e rd^{ec}\right)^{\frac{1}{\psi}} \quad \text{and} \quad e^d = \left(\phi^d_e rd^{ed}\right)^{\frac{1}{\psi}}$$

where e^c denotes the entry rate of firms that initially operate a clean product line and e^d is the rate at which firms operating a dirty product line enter. As with incumbent innovation, entrants become the owner of the highest-quality patent of a random product which is a λ multiple of the level of quality of the previous producer. Without loss of generality, we assume that entrants' innovations are of the type – clean or dirty – in which the innovation was directed.

2.2.3 Creative destruction and productivity growth

Firms that own the patent to produce good j engage in Bertrand competition. As goods are perfect substitutes in the eye of the consumer, the firm with the highest quality attracts all demand. This means that whenever a product is innovated upon, either by entrants or existing firms, the incumbent firm stops producing the product. A multi-product firm that is hit by creative destruction stops producing one of its products. Single-product firms without undeveloped products that see their sole product creatively destroyed exit. This happens at the rate of creative destruction, τ:

$$\tau = \int_0^1 \mathbb{I}_{j \in J_i} \left[x^c_i + x^d_i\right] dj + e^c + e^d. \tag{2}$$

where incumbent innovation is expressed in intensity $x^h_i \equiv X^h_i / n^h_i$ and where J_i is i's product portfolio.

As each creatively destroyed product's quality improves by a multiple λ, aggregate productivity growth in the economy is given by

$$g = \tau \ln \lambda. \tag{3}$$

[7] Evidence of knowledge spillovers both, from past green innovation to current dirty innovations, and from past dirty innovation to current green innovation, can be found for example in Aghion et. al. (2016).

2.3 Financial frictions

There is a financial friction that distorts optimal R&D decisions across firms. When financing R&D, firms are unable to costlessly borrow up to the net present value of their profits. Instead, they face a financing cost $\widetilde{r}(k)$ that depends on the firm's maturity, k. The interpretation of $\widetilde{r}(k)$ is broad: it reflects both credit costs and the implicit costs of being unable to access credit due to credit rationing, as banks may prefer rationing rather than interest hikes to reduce credit supply (e.g. Stiglitz and Weiss 1981). We assume that financing costs are higher for firms with low maturity. The idea is that as firms mature, they build a track record that reduces the monitoring costs of their lenders. The reductions in monitoring costs are then passed-through to the borrowing firm. Firms are born with the lowest level of maturity $k = 1$, and mature step-by-step at Poisson rate ν to higher levels of k over their life time, until reaching full maturity $\overline{k}$.

In the quantitative sections of the paper, we discipline the effect of maturity on $\widetilde{r}(k)$ by directly matching estimates of the differential effect of aggregate financial shocks on innovation for younger firms and mature firms. It is straightforward, however, to microfound the relationship between maturity and credit costs. Assume, for example, a competitive credit market where in equilibrium creditors just break even between the amounts they lend to firms and their expected repayments net of monitoring costs. The source of credit market imperfection is ex-post moral hazard: unless they are adequately monitored, borrowers can escape ex-post repayment, eloping with their borrowed funds. Suppose that to recover the debt repayment on a loan of size L on a firm of maturity level k with probability $\mathscr{P}$ the creditor must incur a monitoring cost equal to $L \cdot c(p,k)$, where $c(\cdot)$ is increasing and convex in $\mathscr{P}$, is decreasing in k, and where furthermore $\partial c/\partial \mathscr{P}$ is also decreasing in k. A functional form that satisfies this is

$$c(\mathscr{P},k) = \left(\frac{\kappa}{\theta(1+k)}\right)(\mathscr{P})^{\theta}.$$

where $\theta > 1$ is the elasticity of monitoring costs in the probability of default. Denoting by r the interest repayment rate incurred by a borrowing firm of maturity k, a competitive credit breaks even if

$$L = \max_{\mathscr{P}} \left\{ pL(1+r) - Lc(\mathscr{P},k)\} \right\}. \tag{4}$$

This leads to the first order condition:

$$1+r = \frac{\partial c}{\partial \mathscr{P}}(\mathscr{P},k) = \left(\frac{\kappa}{1+k}\right)(\mathscr{P})^{\theta-1} \quad \Rightarrow \quad \mathscr{P} = \left(\frac{(1+r)(1+k)}{\kappa}\right)^{\frac{1}{\theta-1}}.$$

Inserting the first-order condition for $\mathscr{P}$ in (4), and isolating the gross interest rate yields

$$1+r = \left[(\theta/(\theta-1))^{\theta-1}\frac{\kappa}{1+k}\right]^{1/\theta} = \widetilde{r}(k).$$

which is clearly decreasing in a firm's maturity k. Note that $\widetilde{r}(k)$ also reflects the tightness of firms' access to credit: the higher $\widetilde{r}(k)$, the lower the firm's credit multiplier (e.g Aghion et al. (1999)). Therefore, since $\widetilde{r}(k)$ is decreasing in k, less mature firms face tighter credit constraints.

2.4 Static optimization

The firm with the leading patent to produce good j faces a unit-elastic demand function $y_{ij}^h = Y/p_{ij}$ as long as, adjusted for quality differences, it sets its price at a lower level than its closest competitor. The profit-maximizing action is thus to limit-price the second-best firm. Because the marginal cost of producing a good depend on whether the producer's patent is dirty or green, the limit price depends on the technology of both the leading and the second-best firm. If both firms have a dirty or clean patent, their marginal costs are equal, and the leading firm charges a markup $\mu^{cc} = \mu^{dd} = \lambda$ over marginal costs. If the leading firm is clean but the producer is dirty, the ratio of their marginal costs is given by

$$\frac{mc^d}{mc^c} = \tilde{p}^o,$$

such that the limit-price markup is

$$\mu^{cd} = \lambda \tilde{p}^o.$$

Conversely, in cases where the leading firm owns a dirty patent and the closest competitor a clean patent, the limit-price markup is

$$\mu^{dc} = \lambda / \tilde{p}^o.$$

It follows that the average markup of producing green goods increase in the relative cost of producing dirty products. An increase in the price of fossil inputs, for example through a carbon tax, therefore raises the markup of firms in green production, while lowering markups of dirty producers.

Inserting the demand function, marginal costs and optimal prices in firms' operating profits, yields the following expression for profits of a firm that uses technology h and competes with a second-best firm that uses technology $\tilde{h}$ Therefore the output-adjusted equilibrium profit on a line in which the incumbent firm uses technology h and the fringe uses technology $\tilde{h}$, is simply given by:

$$\Pi^{h\tilde{h}} = \pi^{h\tilde{h}} Y_t, \quad \text{where} \quad \pi^{h\tilde{h}} = 1 - (\mu^{h\tilde{h}})^{-1}.$$

2.5 Dynamic optimization

Firms choose clean and dirty innovation rates in order to maximize firm value. The value $V(J, k)$ of a firm that produces the portfolio of goods J and that is of maturity level k. The portfolio is divided into products that are implemented (J^I) and unimplemented (J^u), and the total number of unimplemented clean and dirty products is given by $\hat{n}^c$ and $\hat{n}^d$, respectively. The value of a firm is given by the following Bellman equation:

$$
rV_t(J,k) - \dot{V}_t(J,k) = \max_{x^c,x^d} \left\{ \begin{array}{c} \sum_{j\in J^A} \pi^{h_j \tilde{h}_j} Y_t \\ +\sum_{l\in J^U} \zeta \mathbb{E}\left[V_t(J\cup J^A\{h_l h_l'\},k)\right. \\ +\nu\left[V_t(J,k+1) - V_t(J,k)\right]\cdot \mathbb{1}_{\{k<\overline{k}\}} \\ +\tau n^c \mathbb{E}\left[V_t(J\setminus\{ch'\},k) - V_t(J,k)\right] \\ +\tau n^d \mathbb{E}\left[V_t(J\setminus\{dh'\},k) - V_t(J,k)\right] \\ +x^c n^c\left[V_t(J\cup J^U\{c\},k)\alpha + V_t(J\cup J^U\{d\},k)(1-\alpha) - V_t(J,k)\right] \\ +x^d n^d\left[V_t(J\cup J^U\{d\},k)\alpha + V_t(J\cup J^U\{c\},k)(1-\alpha) - V_t(J,k)\right] \\ -w_t \tilde{r}(k)\left[(\phi^c)^{-1}(x^c)^\psi n^c + (\phi^d)^{-1}(x^d)^\psi n^d\right]\right] \end{array} \right\}.
$$

In words: the firm's value is made up of: (i) the sum of instantaneous profits on the lines currently operated by the firm; (ii) the capital gains generated when a firm implements its prior green or dirty innovation, in the former case the number of "green lines" increases by one unit, in the latter case the number of "dirty lines" increases by one unit; (iii) the capital gains induced by maturing by one further step, which can occur only if $k < \overline{k}$; (iv) the capital losses induced by creative destruction on a green line and by creative destruction on a dirty line respectively: in the former case the number of green lines decreases by one unit, in the latter case the number of dirty lines decreases by one unit; (v) the capital gains generated when a firm expands its portfolio of unimplemented innovations by either a green or a dirty patent through innovation; (vi) the cost of green and dirty R&D expenditures.

Proposition 1. *The value V_t of a firm grows at a constant rate g over the balanced growth path*

$$
\dot{V}_t(J,k) = gV_t(J,k)
$$

and can be expressed as the product of aggregate income Y_t and time-invariant products of the number of green and dirty products that the firm produces and the unimplemented innovations they have in stock:

$$
V_t = V_t(k, n^c, n^d, \widehat{n}^c, \widehat{n}^d) = Y_t\left(V^c(k)n^c + V^d(k)n^d + \hat{V}^c(k)\hat{n}^c + \hat{V}^d(k)\hat{n}^d\right)
$$

where t $V^c(k)$ and $V^d(k)$ capture the present value of the profit stream of clean and dirty products, respectively, as well as the present value of the option value of the increase in R&D productivity that comes with producing a clean or dirty good:

$$
V^c(k) = \frac{\mathbb{E}\pi^c + x^c(\overline{k})(1-\alpha)\left(\frac{\zeta}{\rho+\zeta}\right)V^d(\overline{k}) - \omega\tilde{r}(\overline{k})(x^d(\overline{k}))^\psi\phi^{-1} + \nu V^c(k+1)\mathbb{1}_{k<\overline{k}}}{\rho+\tau - x^c(\overline{k})\alpha\left(\frac{\zeta}{\rho+\zeta}\right) + \nu\mathbb{1}_{k<\overline{k}}} \tag{5}
$$

$$
V^d(\overline{k}) = \frac{\mathbb{E}\pi^d + x^d(\overline{k})(1-\alpha)\left(\frac{\zeta}{\rho+\zeta}\right)V^c(\overline{k}) - \omega\tilde{r}(\overline{k})(x^d(\overline{k}))^\psi\phi^{-1} + \nu V^c(k+1)\mathbb{1}_{k<\overline{k}}}{\rho+\tau - x^d(\overline{k})\alpha\left(\frac{\zeta}{\rho+\zeta}\right) + \nu\mathbb{1}_{k<\overline{k}}} \tag{6}
$$

where F is the fraction of goods that is produced using a clean technology while ω is the steady-state labor share. $\widehat{V}^c(k)$ and $\widehat{V}^d(k)$ give the value of owning an unimplemented product innovation, which simply equal the value of owning an implemented product discounted by the expected time to implementation:

$$\widehat{V}^c(k) = \left(\frac{\zeta}{\rho+\zeta+\nu\mathbb{1}_{k<\bar{k}}}\right)V^c(k) + \left(\frac{\nu}{\rho+\zeta+\nu}\right)\widehat{V}^c(k+1)\mathbb{1}_{k<\bar{k}},$$
$$\widehat{V}^d(k) = \left(\frac{\zeta}{\rho+\zeta+\nu\mathbb{1}_{k<\bar{k}}}\right)V^d k) + \left(\frac{\nu}{\rho+\zeta+\nu}\right)\widehat{V}^d(k+1)\mathbb{1}_{k<\bar{k}}.$$

This value determines optimal innovation intensities for clean and dirty innovation, which are constant across firms of the same age k:

$$x^h(k) = \left(\phi^h \frac{(1-\alpha)\widehat{V}^h(k) + \alpha\widehat{V}^{\tilde{h}}(k)}{\omega\psi\tilde{r}(k)}\right)^{\frac{1}{\psi-1}}. \tag{7}$$

Optimal entry rates are similarly given by

$$e^h = \left(\phi^h \frac{\widehat{V}(1)}{\omega\psi\tilde{r}(1)}\right)^{\frac{1}{\psi-1}}. \tag{8}$$

Proof: Appendix B.

The proposition shows that a firm's innovation rates increase in the value of becoming a good's producer, which is the sum of the discounted value of the good's profits and the option value of the firm's additional capacity for research and development. The optimal innovation rates declines in the cost of R&D, which is a combination of the cost of hiring researchers and the cost of financing research activities.

2.6 Maturity, size and technology-type distributions

The fraction of goods that is produced by firms with maturity level k and using technology h, is given by

$$M^h(k) = \frac{\sum_{n^c,n^d} M_{n^c,n^d,a} \times n^h}{\sum_{1\le k\le\bar{k}}\sum_{n^c,n^d} M_{n^c,n^d}(k)\times(n^c+n^d)},$$

where $M_{n^c,n^d}(k)$ denotes the measure of the set of k-firms that produce n^c clean goods and n^d dirty goods. This fraction is important, as the total number of patents that a firm is expected to come up with is proportional to the number of clean and dirty products that each firm produces. It follows that aggregate creative destruction is determines by the product of the innovation rates of firms of maturity k, and their share in the total mass of firms. Indeed, in the aggregate rate of creative destruction (2), the first term can be written as:

$$\int_0^1 \mathbb{1}_{j\in J_i}\left[x_i^c + x_i^d\right]dj = \sum_{k=1}^{\bar{k}}\left(M^c(k)x^c(k) + M^d(k)x^d(k)\right).$$

Note, furthermore, that the aggregate share of products hat are produced using the green technology in equilibrium (F) is also straightforward to calculate from $M_{n^c,n^d}(k)$

$$F = \frac{\sum_{1\le k\le \bar{k}} \sum_{n^c,n^d} M_{n^c,n^d,a_k} \times n^c}{\sum_{1\le k\le \bar{k}} \sum_{n^c,n^d} M_{n^c,n^d,a_k} \times (n^c + n^d)}. \tag{9}$$

Appendix C describes how we compute the balanced growth path values of $M_{n^c,n^d}(k)$. In brief, we use the fact that this mass changes as a result of innovation, creative destruction and maturity, placing all flows in $M_{n^c,n^d}(k)$ in a transition matrix. We then use the fact that $M_{n^c,n^d}(k)$ is constant for all n^c, n^d and k along the balanced growth path to derive the equilibrium firm size and maturity distribution.

2.7 Aggregate variables and labor market equilibrium

We now characterize the economy's aggregate variables. The equilibrium wage is given by

$$w = \exp\left(\int_0^1 \ln q_j \, dj\right)\left(\exp\left(\overline{\ln\mu^c}\right)^F \exp\left(\overline{\ln\mu^d}\right)^{(1-F)}\right)^{-1}, \tag{10}$$

where the first term is the standard CES productivity index, while the variables in the second term are

$$\overline{\ln\mu^c} = F\ln\mu^{cc} + (1-F)\ln\mu^{cd} \quad \text{and} \quad \overline{\ln\mu^d} = F\ln\mu^{cd} + (1-F)\ln\mu^{cc} + \ln\tilde{p}^o.$$

The second term in (10)captures the degree to which wages are suppressed by the fact that firms have market power and thus charge markups, and the fact labor is combined with other inputs in the production of dirty products. Aggregate output is given by:

$$Y = L^p \exp\left(\int_0^1 \ln q_j \, dj\right)\mathcal{M} \tag{11}$$

where L^p is a composite of production labor and the dirty production input. Derivations are provided in Appendix A. The term $0 < \mathcal{M} \le 1$ measures the productivity loss that arises when firms have heterogeneous markups, because products that sell at a high markup are demanded in inefficiently low quantities. A term similar to the latter is derived in Peters (2020).

Finally, the household's utility function implies that is supplied inelastically by households at a measure standardized to 1. Equilibrium on the labor market therefore requires that

$$1 = L^d + L^c + L^{rd} + L^e, \tag{12}$$

where L^c and L^d is the labor used to produce clean and dirty intermediate goods while L^{rd} and L^e denote the labor required for innovation by incumbents and entrants. The latter two are given by:

$$L^{rd} = \sum_{1\le k\le \bar{k}} \left(M^c(k)\left[x^c(k)\right]^{\psi}(\phi^c)^{-1} + M^d(k)\left[x^d(k)\right]^{\psi}(\phi^d)^{-1}\right)$$

$$L^e = \left[e^c\right]^{\psi} (\phi^{ec})^{-1} + \left[e^d\right]^{\psi} (\phi^{ed})^{-1}$$

For labor involved in the production of clean products, demand equals $L^c = FY$, as only labor is used in the production of these goods. Labor demanded for the production of dirty products is found through the production function for dirty products, in combination with labor market equilibrium condition (12).

2.8 Equilibrium definition

Definition 1. *The economy is in a balanced growth path equilibrium if for every t and for every firm maturity level k, the variables e^c, e^d, τ, r, L^p, $\mathcal{M}$ and the functions $x^c(k)$, $x^d(k)$, $V^c(k)$, and $V^d(k)$ are constant while Y and w grow at the constant rate g that satisfies (3) with creative destruction rate (2). The value of owning clean and dirty product lines are respectively given by (5) (6), while innovation rates for incumbents and entrants are respectively given by (7) and (8). The equilibrium fraction of green products F is given by (9). The interest rate obeys the Euler equation, output and wage are respectively given by (10) and (11). Appendix A provides expressions for L^p, using labor market clearance (12) and markup dispersion $\mathcal{M}$.*

2.9 Discussion: financial disruptions during a green transition

In the following sections, we show that a tightening of financial conditions that raises the costs of innovating for young firms will particularly affect green innovation - if the economy is in a green transition. We define a green transition as a phase during which the fraction of goods produced with green technologies, F, is increasing. F increases whenever the sum of incumbent and entrant green innovation rates exceeds the number of green products that is creatively destroyed, τF.

A green transition starts after the relative cost of producing with a polluting technology, $\tilde{p}^o$, increases. This lowers the markup of producing dirty products and raises the markup of producing clean products. The optimal clean innovation rates for incumbents and entrants thus rise, while optimal dirty innovation rates falls. $\dot{F}_t$ exceeds 0 and will stay positive until the economy has transitioned to its final value of F.

Early in the transition, young firms are responsible for a larger share of green innovation than along the balanced growth path. This is because of path-dependence: total green innovation by firms of a given maturity is equal to their green innovation rate multiplied by the fraction of products in the economy that is already produced using green technologies, $M^c(k)$. As incumbents produce largely with polluting technologies – $M^c(k)$ is low – the immediate rise in green innovation at the start of the transition is largely driven by entrants. In subsequent years those entrants expand and become part of the incumbents, but their average maturity is initially below that of the incumbents that were active prior to the transition.

The below-average age of firms that innovate in green technology makes the green transition susceptible to financial shocks. We model financial shocks as an increase in κ, which causes an increase in financing costs $\tilde{r}$ that is larger for young firms than for old firms. Note that the elasticity of all innovation rates in with respect to $\tilde{r}(k)$ is equal to $\frac{1}{1-\psi}$ - regardless of whether the innovation is dirty or clean. The effect of an increase in κ on total green innovation $M_t^c(k)x_t^c(k) + e_t^c$ is thus endogenously larger than the effect of κ on total dirty innovation.

3 Green innovation and financial tightness in the data

In this section we provide supporting evidence on the causal relationship between financial conditions and green innovation. In particular, we use heterogeneous exposure of firms to tight credit at the time of the Global Financial Crisis as a natural experiment to provide evidence of a negative effect of credit tightening on green innovation. To do so, we combine firm-level measures of green versus non-green innovation from IPC patent classification codes with data on firms' exposure to tight credit during the Global Financial Crisis. From this data we establish three stylized facts that are consistent with the model: (1) green innovation is declines significantly as a result of tight credit while non-green innovation does not; (2) the negative effect of tight credit on green innovation is concentrated in young firms; and (3) young firms generally were more likely to engage in green innovation than older firms prior to the crisis.

3.1 Strategy

To establish these stylized facts we analyze the effect of exposure to the 2008-2009 Global Financial Crisis for German firms. We use the fact that firms that relied on Commerzbank were disproportionately subject to a tightening of their credit constraint, as shown in Huber (2018).[8] Commerzbank was a large German bank prior to the crisis, with a market share of around 9% in 2006. It incurred significant trading losses on the U.S. subprime market in 2008, causing the bank's equity to fall by 68%. These losses forced Commerzbank to cut its assets significantly. As Basel II regulation assigns a significant risk weight to corporate loans, reduced credit supply to corporations are an effective way of doing so. As discussed by Huber, this led Commerzbank to reduce its lending stock by 35% between 2006 and 2013, which is over three times the average cut in lending stock by German banks after the Global Financial Crisis.

We use the fact that firms were differentially exposed to Commerzbank's lending cut to obtain a measure of differential exposure to the Global Financial Crisis. Given the long-term nature of relationships between firms and banks, induced by the fact that repeated interaction improves the ability of banks to screen and monitor lenders (e.g., Boot 2000), this means that firms that relied on credit from Commerzbank prior to the Global Financial Crisis faced a greater tightening of their credit constraint. Thus, we conjecture that green innovation should be more negatively affected by the crisis in those German firms that were more heavily dependent upon Commerzbank lending prior to the crisis.

To measure pre-crisis dependence on Commerzbank for any firm i in our sample, we use the fraction of a firm's pre-crisis relationship banks that is a branch of Commerzbank. Relationship banks are the primary banks that firms obtain banking services from, and are based on firms' respond to surveys from credit rating agency Creditreform in 2006. We then look at how firm-level flows of green and non-green patents depend upon the interaction between year effects and the above firm-level measure of pre-crisis Commerzbank dependence. More specifically, we perform a difference-in difference estimation to assess the extent to which green and dirty patenting react differently to the Global Financial crisis in firms with high versus low pre-crisis dependence upon Commerzbank lending.

[8] Huber (2018) looks at the effects of the Global Financial Crisis on employment, investments and overall patenting. Its Online Appendix B provides a detailed analysis of the trading losses incurred by Commerzbank and the effect this had on lending.

Table 1: Descriptive Statistics

	Mean	St. Dev.	Median	10th Pct.	90th Pct.
Value added	73,384,421	6.14E+08	11,249,385	3,342,691	97,015,680
Employment	969.3	10831.9	144	35	1145
Operating Profits	73384413	6.14E+08	11249381	3342692	97015688
Fixed Assets	61749246	6.2E+08	6091660	622625.9	96746492
Wage Bill	28111845	2.72E+08	3912469	1055326	36397191

Notes: Summary statistics for the merged Dafne-Creditreform-Patstat dataset.

More formally, we estimate the Pseudo-Poisson Maximum Likelihood equation:

$$E_t[y_{it} \mid CB_i, i, t] = \exp(\gamma_t \mathbb{1}_t CB_i + \phi_i + \psi_t), \tag{13}$$

where y_{it} denotes firm i's flows of green and dirty patents in year t, $\mathbb{1}_t CB_i$ is the interaction term between year t and firm i's pre-crisis Commerzbank dependence index CB_i. We expect the coefficients γ_t to be negative in the aftermath of Commerzbank's 2008-2012 reduction in credit supply. Exposure is given by

$$CB_i = \sum_{h \in H_i} \frac{\mathbb{1}_{h=CB}}{|H_i|},$$

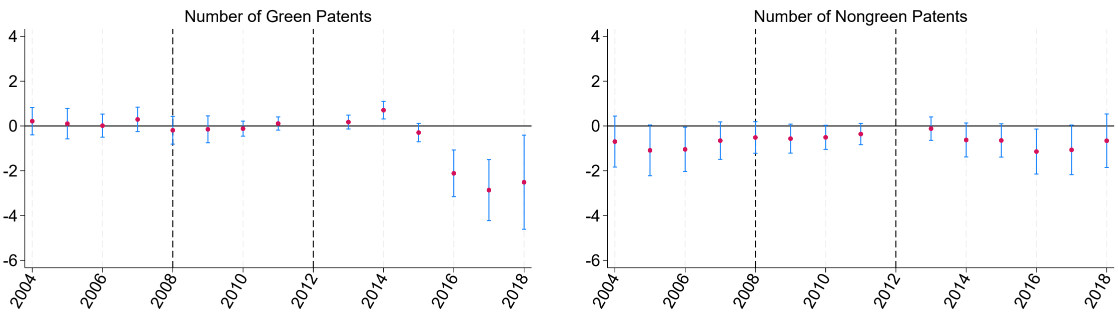
where H_i are the set of relationship banks that firm i reports in 2006, while $\mathbb{1}_{h=CB}$ is the indicator function that equals 1 if h is a Commerzbank branch.

3.2 Data

We combine firm-level accounting data with data on exposure to the Global Financial Crisis and data on patenting. The accounting data comprises panel balance sheet and income statement information from Dafne, which is the German subset of Bureau van Dijk's (BvD) Orbis. Patenting data comes from the 2018 iteration of PATSTAT, containing world-wide coverage on patents from over 90 patent offices, which we merge to Dafne using the BvD identifier. Finally, we obtain firms' primary lenders from data obtained by credit rating agency Creditreform. We restrict ourselves to the subset of firms that has at least filed for one patent over the firm's history by 2018, and to firms for which we have continuous non-missing accounting data on employment, wages, loans, value added and fixed assets between 2007 and 2012. The resulting sample covers 18 years of data for 601 firms. Summary statistics are provided in Table 1.

To measure green innovation we classify firms' patent filings into green and non-green using the OECD classification proposed by Haščič and Migotto (2015). They propose a categorization of patents based on the international patent classification (IPC) code. based on the European Patent Office's Y02 classification. Green patents fall under four technology fields: environmental management technologies, water-related adoption technologies, biodiversity protection technologies, and climate change mitigation technologies, where the latest makes up the largest share of green IPC codes. We classify patents as green when at least one of a patent's IPC codes is included in the OECD classification, and define a firm's patent count as the number of (non)-green patents it applied for within a calendar year.

Figure 2: Effect of Global Financial Crisis on Patent Applications



(a) Green Patents (b) Non-Green Patents

Notes: The figure plots the effect of exposure to Commerzbank on non-green patents (left) and green patents (right). Patents are classified as green if at least one IPC code relates to environmental management, water-related adaption technologies, biodiversity and ecosystem health, climate-change mitigation (OECD classification - Haščič and Migotto 2015). Confidence bounds are at the 95% level based on firm-clustered standard errors.

3.3 Results

We estimate the PPML regression in equation (13) for various samples. In a first estimation, we include all firms in the Dafne-PATSTAT dataset of German innovative firms, using counts of either green or non-green patent as the dependent variable. Figure 2 presents the results. The left-hand figure plots the effect of greater exposure to the Global Financial Crisis through Commerzbank on green patents and the right-hand figure does the same for non-green patents. Confidence bounds are at the 95% level using firm-clustered standard errors. The figure shows that exposure to the crisis has no meaningful effect on innovating in non-green technologies, whereas it had a significantly negative effect on green innovation. The point estimates are large: the elasticity of green patenting with respect to Commerzbank dependence is about 3 by 2018. Note, however, that few firms rely solely on loans for Commerzbank – the average value of CB_i is 0.17 with a 90th percentile of 0.5. A firm with an average (90th percentile) dependence on Commerzbank will therefore have 28% (45%) fewer green patents than a firm that has no direct exposure.[9]

Figure 2 suggests that there is a substantial lag in the effect of the Global Financial Crisis on green patenting. Indeed, previous work including Chodorow-Reich (2014) and Duval et al. (2017) finds more immediate effects of exposure to tight credit on variables such as investment and employment. To understand the slower effect of the crisis on green patenting by German firms, two facts are relevant. First, German firms with close ties to Commerzbank faced a protracted restriction in their credit supply. The stock of Commerzbank's lending stock starts shrinking faster than that of its German peers in 2008, but the gap widens until 2012. Second, there is a lag between innovative investments such as research and development, and realized innovation in the form of patents. We observe that the size (as measured by capital or employment) of more exposed firms starts shrinking earlier than patents. The decline in

[9]Despite the modes sample size, the results in Figure 2 are robust to the inclusion of detailed fixed effects. In Figure A3 in Appendix A we show that the effect of the crisis on both green and non-green patents is qualitatively and quantitatively robust to the addition of additional controls such as size-year fixed effects and detailed industry-year fixed effect.

exposed firms' green patenting from 2015 therefore seems consistent with the plausible lag between reduced credit supply and a shortfall in realized innovation.

We next show that the negative effect of exposure to Commerzbank on green firms is driven by *young* firms. To do so, we split our sample by a measure of pre-crisis firm age. Young firms are more likely to be impacted by financial constraints, as they have not accumulated a sufficient track record nor equity to borrow the optimal amount for innovative investments (e.g. Moll 2014). They are also less likely to have access to bond or equity markets, or use cash flow to cross-subsidize green innovation (e.g. Ivashina and Scharfstein 2010, Chodorow-Reich 2014, Xiao 2022).

We estimate the following "triple difference" regression:

$$\mathbb{E}[y_{it}] = \exp\left(\gamma[CB_i \times Post_t \times Younger_i] + \beta[CB_i \times Post_t] + \alpha[Younger_i \times Post_t] + \phi_i + \psi_t\right).$$

The key outcome is green innovation and the key coefficient is γ which indicates whether the impact of financial crises are greater for young firms. We include firm and time dummies and saturate the model with pairwise fixed effects, so we identify purely from the "triple difference" term.

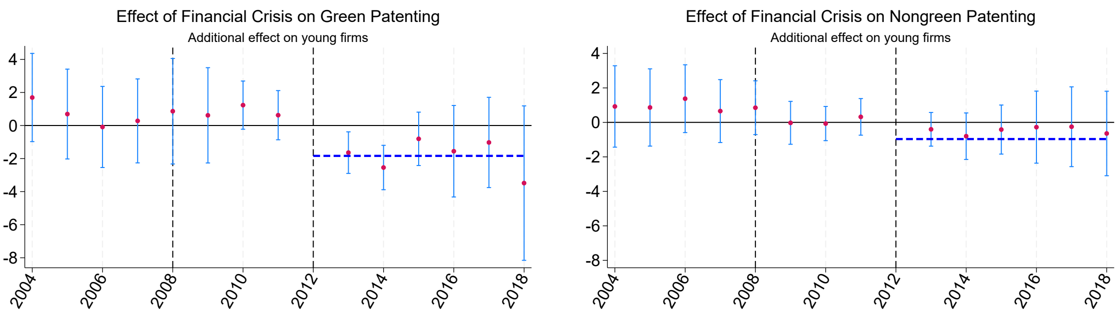
Table 2: Effect of Commerzbank Dependence on Patenting

	(1)	(2)	(3)	(4)	(5)	(6)
	Green	Nongreen	Total	Green	Nongreen	Total
Young x				-1.836***	-0.967	-1.019**
CB_Dependence x Post				(0.641)	(0.589)	(0.458)
Post-2012 x	-0.506***	-0.107	-0.236	0.950*	0.303	0.369
CB_Dependence	(0.166)	(0.334)	(0.229)	(0.555)	(0.474)	(0.418)
Young x Post				0.679*	0.362*	0.381**
				(0.364)	(0.186)	(0.175)
Observations	2,700	7,080	7,260	2,700	7,080	7,260

Notes: PPML regressions of patent counts. "Post" is a dummy for a year after 2012. Firm-clustered standard errors are in parentheses. "CB_Dependence" indicates the measure of pre-2007 dependence on Commerzbank. All columns include firm and year fixed effects.

Table 2 presents the results. Column (1) presents the restricted version of the regression specification implicitly underlying the previous event study figure. There is a significant negative impact of the crisis on Commerzbank related firms on green patents. By contrast, column (2) shows that although there is also a negative impact on non-green patents, the coefficient is only a fifth of the size and insignificant at conventional levels. Column (3) pools them all together and shows a coefficient in between the previous two columns. The last three columns implement our preferred model which allows the treatment effect to vary with firm age. Column (4) shows that the negative effect of the financial crisis on green patents is significantly larger for young firms than for old firms. Column (5) shows that there is also a more strongly negative effect of the crisis for the non-green patents (as the model would suggest), although the

Figure 3: Conditional Effect of Firm Age on Patenting Applications



(a) Green Patents

(b) Non-green Patents

Notes: The figures plot the conditional effect of young firm exposed to Commerzbank on green and non-green patenting. The point estimates from Table 2 are shown as a blue dashed line for clarity. Patents are classified as green if at least one IPC code relates to environmental management, water-related adaption technologies, biodiversity and ecosystem health, climate-change mitigation (OECD classification - Haščič and Migotto 2015). Confidence bounds are at the 95% level based on firm-clustered standard errors.

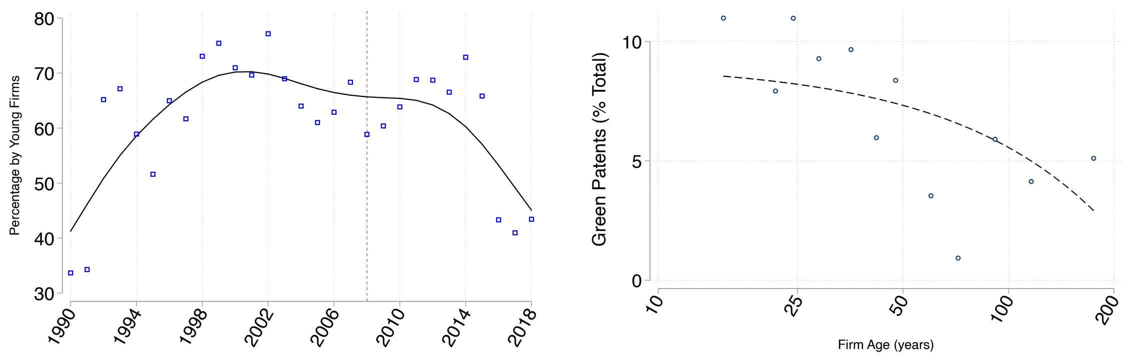
coefficient on this interaction is half the size of column (4), and is insignificant. The final column shows that overall innovation falls signficiantly after the crisis for exposed young firms.

Figure 3 visualizes this difference plotting the more general event study allowing the coefficient on the interaction between Commerzbank exposure and young firms to be different in every year. There is a clear negative impact in the years following the crisis for green innovation, but not for non-green innovation, with no sign of pre-trends in either case.

Given that young firms appear to drive the negative effect of crisis-exposure on green patenting, we next assess the importance of younger firms for overall green innovation. Out of 7,998 green patents by firms in our sample from 1990 to 2018, firms that are classified as young in 2006 are responsible for 5127 patents, or 64%. To the contrary, of the 53,684 non-green patents in our sample, young firms are responsible for only 11,480 patents, or 21%. Young firms therefore play an out-sized role in green innovation. Figure 4a shows how the importance of young firms in green innovation changes over time. At the start of the sample in 1990, the contribution of firms that are young in 2006 is modest, as these firms are small or yet to be born. Their contribution grows over time and peaks in the mid-2000s. After the crisis, their contribution to overall green innovation rapidly declines, falling from 70 to 40% of green patents.

Figure 4b provides a further illustration of the green innovation behavior of young firms. It presents the binned scatter plot of a firm's fraction of patents that is green versus the firms age, using data up to 2013. It shows that there exists a near monotonic relationship between age and green innovation: the younger the firms, the greater is the fraction of their patents that is green. The difference is large: the share of green patents in total firms is more than twice as large for the youngest than for the oldest firms.

Figure 4: Younger Firms and Green Patenting



(a) Share of Green Patents - Younger Firms (b) Share of Green Patents - By Age

Notes: The left-hand figure plots the percentage of green patents that is filed by firms with a below-median age in 2006. Scatters give data, the solid line is gives the HP filtered trend using a smoothing parameter of 100. The right-hand figure presents a binned scatter plot with the average percentage of patents in a bin that is green on the vertical axis, and age on the horizontal axis.

4 Green innovation and a contraction in monetary policy

The adverse impact of a contraction in credit supply on green innovation extends beyond financial crises. In practice, central bank interest rate increases are a common source of credit tightening, and we demonstrate that these increases disproportionately reduce green patenting compared to non-green patenting. This finding complements our micro-level estimates by capturing the aggregate-level response of green patents to monetary policy, thus accounting for general equilibrium effects that our difference-in-differences estimates miss.

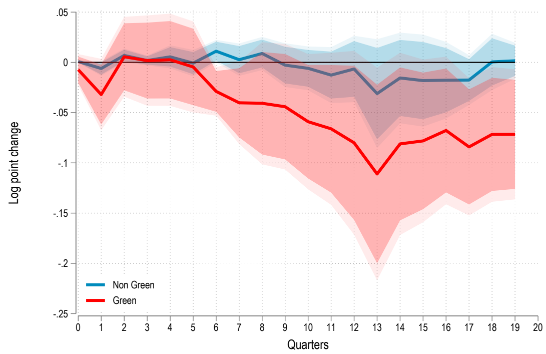
To quantify the negative impact of monetary policy tightening, we estimate the impulse response function to monetary policy shocks as estimated by Kerssenfischer (2022). These shocks quantify the surprise in Euro Area monetary policy announcements by measuring changes in bond yields in narrow windows around monetary policy announcements by the European Central Bank. Our dataset spans from March 2002 to June 2023.[10] Denoting these shocks as S_t, we estimate the following equation:

$$Y_{t+h} - Y_t = \alpha_h + \beta_h S_t + \gamma_h' Z_t + \varepsilon_t \tag{14}$$

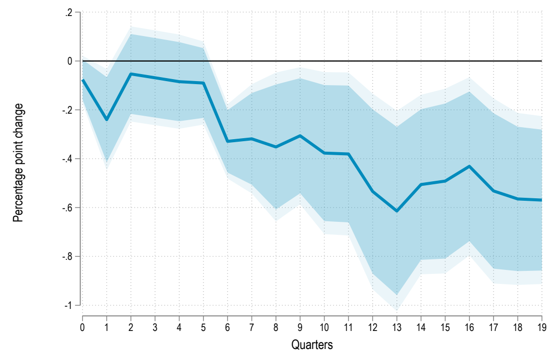
Here, Y_t represents the dependent variable, and Z_t includes a standard set of control variables as recommended by Jordà (2005), encompassing four quarters of lagged dependent variables and lags of the monetary policy shocks. We conduct separate local projection estimates for each horizon h from 0 to 20 and graph the sequence of β_h coefficients. The dependent variable is measured using the application dates of patents granted by the European Patent Office.

[10] Deviations of monetary policy announcements from market expectations may also be due to the fact that the European Central Bank has information about the stance of the economy that markets do not share. The monetary policy shocks in Kerssenfischer (2022) adjust for this informational effect through sign restrictions.

Figure 5: Effect of Monetary Policy Shock on Green and Non-Green Innovation



(a) Green and Non-Green Patents (log)



(b) Green Patents as % of Total

Notes: The figure plots the β in (14) for each horizon. Confidence bounds are at the 90 and 95% level.

Figure 5 presents the results. The left-hand figure presents the effect of a one-standard deviation monetary policy shock on the log of the number of dirty (blue line) and green (red line) patents. The figure shows a significantly negative effect of monetary policy shocks on the number of green patents issued. The coefficient is large: the trough of the log-change in the number of green patents reaches -0.11. The effect of monetary policy on dirty patents is smaller: the decline in patenting is at most 0.02 log points and is not statistically significant at conventional levels. The right-hand figure shows a significant effect of the monetary shock on the ratio of green patents to overall patents. After a one-standard deviation monetary policy shock, this ratio declines to a trough of 0.6 percentage points, which happens three years after the incidence of the monetary policy shock. This decline is large: the highest ratio of green patents to total patents which we observe in the data is 10%, which means that a one-standard deviation monetary policy shock reduces the green patent share by 6%.

Discussion The empirical analysis has presented four results on the relationship between financial constraints, green innovation, and firm dynamics. Exposure to the Global Financial Crisis has a significantly negative effect on firms' green innovation, but not on non-green innovation. Similarly, green patents fall as the result of monetary policy shocks, while dirty patents do not. This effect appears to be driven by younger firms, which are on average smaller. These firms are generally responsible for a significant fraction of all green innovation in the run up to the Global Financial Crisis. These findings are consistent with our fully-fledged model of the interplay between financial constraints, green versus dirty innovation, and firm dynamics. We now quantify that model in the next section.

5 Quantification

Before turning to the analysis of the transition path towards a green economy, in this Section we calibrate the 9 parameters that govern the model's properties in the initial and final steady state. For 8 parameters we use the same values in the initial and final steady state: innovation productivity shifters ϕ^c and ϕ^d, entry productivity shifters ϕ_e^c and ϕ_e^d, directed innovation spillover parameter α, as well as cost elasticity ψ, innovation step λ, and the discount rate ρ. For the remaining parameter, the relative cost of dirty production $\tilde{p}^o$, we change the value over time as it is the main policy parameter that governs the transition to clean technology.

5.1 Initial Balanced Growth Path

We calibrate ρ and ψ externally. The discount rate ρ is set to 0.02, which is a standard value in the literature (e.g. Akcigit and Kerr (2018), Acemoglu et al. 2018, Aghion et al. 2023b). This value delivers a real interest rate of 2.6% along the balanced growth path if total factor productivity grows at 1.6%.

We set the curvature of R&D costs ψ to 2.6. This parameter can be inferred from the elasticity of R&D with respect to the user costs ($\epsilon_{x,w}$) in the following way:

$$\psi = -\frac{\epsilon_{x,w} - 1}{\epsilon_{x,w}}.$$

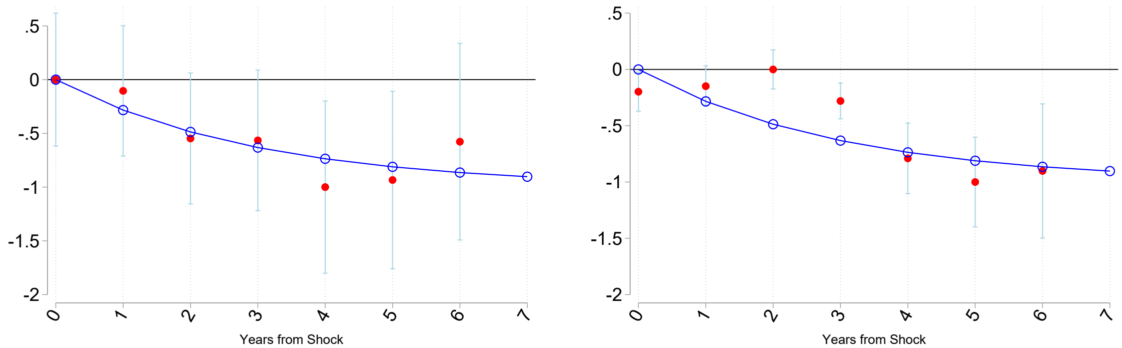
A long literature has studied this elasticity using micro data on firms' response to R&D tax credit. A recent analysis of micro data on R&D tax elasticities across OECD countries in Appelt et al. (2022), which finds that elasticities are closer to -0.6. This implies a calibration of $\psi = 2.00$. Common values of the elasticity, as discussed by Bloom et al. (2002) range from -0.5 to -1.0, which also implies a calibration of ψ between 2 and 3. We set $\psi = 2$, which is the value used in Acemoglu et al. (2018) and Akcigit and Kerr (2018).

The innovation step λ determines average markups, and thus the profit share in the economy's GDP. We set λ to 1.1 to achieve a profit share of around 9%, consistent with estimates in Barkai (2020).

To calibrate the probability that a firm develops a patent in the non-directed technology, α, we target the probability that a single-patent firm innovates in dirty (green) innovation when its initial patent is green (dirty). The ratio of this probability and the probability that the firm innovates in its original technology corresponds directly to α in the model. We calculate the fraction among single-patent German firms between 1985 and 1995, and find a value for α of 0.049.

We set the rate at which innovations are implemented, ζ, to 1/3. This means that there is on average a three-year lag between a finance-induced reduction in R&D investments and the subsequent fall in patenting. This lag aligns with the long literature that estimates empirical lags between R&D and innovation (going back to Pakes and Griliches 1980), which finds lags of up to five years. The resultant impulse response for innovation in the model is furthermore consistent with our own micro estimates in Section 3. Figure 6 plots the fraction of full the effect of a financial shock on patents that has been achieved each year after the shock, and shows that the model matches the timing of the effect on patenting well.

Figure 6: Lagged Effect of Financial Shock on Patenting: Model versus Data



(a) Non-Green Patents (b) Green Patents

Notes: The figure compares the post-shock impulse responses of exposure to Commerzbank (Figure 2, red scatters) to the effect of a financial shock on patenting in the model (blue-solid lines). In order to solely affect the model's ability to match the lagged effect of the shock, the axes are normalized such that the lowest coefficient is zero, and the peak effect is -1. As ζ gives implementation lags for both green and dirty innovation, the model lags are equal in both plots.

The remainder of the parameters are structurally estimated, for which we rely on moments from macro data and from the micro data on German firms in Section 3. The structural estimation chooses the set of parameters that satisfies the objective function:

$$\min \sum_{k=1}^{3} \frac{|\,\text{model}_k - \text{data}_k\,|}{(|\,\text{model}_k\,| + |\,\text{data}_k\,|)\,0.5}, \tag{15}$$

where model_k and data_k respectively refer to the simulation and data for moment k. Moments carry equal weight. We use the genetic algorithm to minimize the loss function, as detailed in Appendix C.

We assign equal R&D productivities for both dirty and clean innovation. This means that any difference in patenting on these technologies comes from the relative cost of owning a clean patent, which in turn is driven by $\tilde{p}^o$, and by financing costs. To calibrate the R&D productivity scalars for incumbents and

Table 3: Quantification Strategy Initial Balanced Growth Path

Parameter	Description	Target	Approach
ψ	R&D cost curvature	R&D cost elasticity	External
ρ	Discount rate	Risk-free interest rate	External
λ	Innovation step size	Profit share in GDP	External
α	Innovation spillover	Patenting specialization	External
ζ	Implementation lag	Patenting impulse response	External
$\phi^d = \phi^c$	R&D productivity scalar	Productivity growth	Indirect inference
$\phi^d_e = \phi^c_e$	Entry cost scalar	Entrants as % of firms	Indirect inference
$\tilde{p}^o$	Relative dirty cost	Relative value of green patents	Indirect inference

Notes: The structural estimation chooses parameter values in order to minimize the distance between theoretical and empirical moments along (15). The initial balanced growth path is disciplined using moments derived from 1985-1995 data.

Table 4: Parameter Values and Performance on Targeted Moments

Parameter	Target	Empirical Target	Theoretical Moment	Parameter Value
ψ	R&D cost elasticity	-1.00	-1.00	2.00
ρ	Risk-free interest rate	3.6%	3.6%	0.02
λ	Profit share in GDP	9.0%	9.0%	1.10
α	Patenting specialization	95.1%	95.1%	.951
ζ	Average lag between R&D and patent filing	3.00	3.00	1/3
$\phi^d = \phi^c$	Productivity growth	1.60%	1.60%	.12
$\phi_e^d = \phi_e^c$	Entrants as % of firms	11.4%	11.4%	.23
$\tilde{p}^o$	Share of green patents in total patents (%)	10.6%	10.6%	.93

Notes: The structural estimation chooses parameter values in order to minimize the distance between theoretical and empirical moments along (15). The initial balanced growth path is disciplined using moments derived from 1985-1995 data.

entrants we use a macro moment to discipline the dirty technology parameters, and then rely on relative patenting rates of dirty and clean technology in order to discipline the clean technology parameters. We discipline the dirty R&D productivity parameter by targeting the growth rate of German total factor productivity between 1985 and 1995, which is 1.6%. To discipline the efficiency of R&D by entrants, we target an entry rate of 11.4%, in line with the German average from 1985 to 1995.[11]

The final parameter to be disciplined is the relative cost of producing a dirty product in the initial steady state, $\tilde{p}^o$. This parameter determines the relative profitability of producing a clean product compared to a dirty product. We discipline this parameter by targeting the fraction of all patents that is clean in the German data from 1985 to 1995, which is 10.6%.

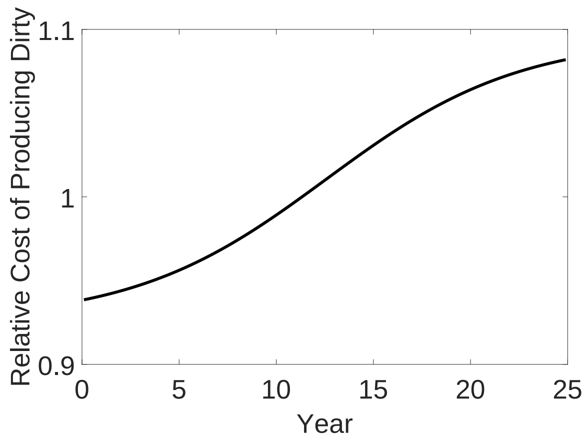
The resulting parameter values are presented in Table 4. The model is able to match the aggregate R&D intensity, green innovation rates by entrants and incumbents, total factor productivity, and the relative value of green and dirty patents with precision. The model overestimates the percentage of firms that specializes in green or dirty innovation, as well as the overall rate of entry.

5.2 Final Balanced Growth Path

In the final steady state we leave all parameters unchanged, except $\tilde{p}^o$. The relative cost of dirty production is the variable that we adjust to summarize policy-induced drivers of the green transition. As our focus is on the interaction between finance and green innovation, we refrain from detailing the type of interventions that policy makers may enact in order achieve a reduction in total emissions. To calibrate $\tilde{p}^o$ in the final steady state, we are guided by the fact that a 65% decline in emissions that is needed by 2035 (compared to 2010) to limit global warming to 1.5 degrees Celsius (IPCC 2023). Given that all firms in the model are subject to the same factor prices for their respective technologies, pollution in the model is proportional to the fraction of goods that is produced with the dirty production technology. In the initial steady state with parameters from Table 4, 89% of goods are produced using the dirty technology. Setting $\tilde{p}^o$ to λ gives a steady state in which 32% of goods are produced with the dirty technology – which is a 64% reduction.

[11] In lieu of data on business dynamism for these years, we calculate this entry rate as the proportion of firms that files for a patent for the first time over firms with a patent history by that respective year.

Figure 7: Path of the Cost of Producing with Polluting Technology



Notes: The figure plots the path of the relative cost of producing with a polluting technology, $\tilde{p}^o$.

5.3 Calibrating the transition and the financial shock

In this subsection we derive the benchmark transition path towards a green economy induced by an increase in $\tilde{p}^0$ - e.g. as induced by an increase in emission tax. Figure 7 plots the path of the costs of producing with the clean technology, which rises gradually from 0.91 to 1.10. That is, polluting production of a good was around 10% cheaper than producing a clean product at the start of the policy, and gradually increases to be around 10% more expensive. In the steady state, this induces a reduction of emissions by around 64% of the initial steady state. We discipline the speed of transition such that we match the empirical rise of green patenting after the agreement of the December 1997 Kyoto Protocol. Figure 8 shows the accelerated rise in the fraction of products produced with the green technology between year 0 and year 10 as implied by our calibrated model. Figure 9 shows that growth in the share of green innovation in the model (left-hand figure) between year 0 and year 10 is in line with the increase in the share of green patenting in total patenting in the data between 1998 and 2008 (right-hand side). In both the data and the model, there is about a 120% increase in patenting over this horizon.

As younger firms are affected by the Global Financial Crisis, a further parameter that requires careful calibration is the rate at which firms mature. In line with the empirical analysis, we separate firms into two categories (young and mature) and set the rate at which firms mature (ν) equal to 0.12. This yields that 10 years after the start of the transition, 70% of clean innovations come from young firms. This is consistent with the share of patents coming from younger firms just prior to the Global Financial Crisis, as it appeared in the above Figure 4.

6 A financial shock during the green transition

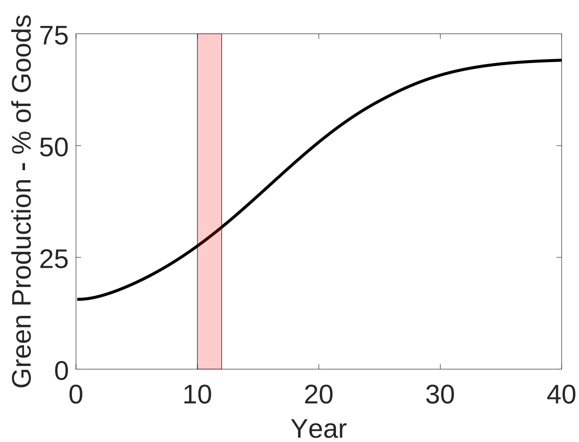
We now analyze how a financial shock affects the green transition path. More specifically, we look at the effect of an increase in financing costs, κ, modeled so as to match the decline in new loans during the Global Financial Crisis. The increase in κ raises the financing costs of younger and therefore less mature firms more than it raises the financing costs of older and hence more mature firms. The increase in financing costs occurs 10 years after the start of the green transition, when young firms are responsible for the majority of green innovations.

Our calibration is informed by two empirical moments. First, we target the decline of credit supply to German firms during the Global Financial Crisis. To do so, we target the change in credit issued to non-financial corporations from the World Bank's Global Financial Development indicators. Second, we use the fact that our empirical evidence shows that exposure to the crisis only affected patenting for younger firms. We therefore load the increase in credit costs solely on firms in the "young" maturity status, $k = 1$.

The path of new bonds and credit to German firms is plotted in the right-hand panel of Figure 10. The figure plots an index of the sum of syndicated loans and bonds to German firms as a percentage of GDP. New loans fell substantially at the onset of the crisis, with a maximum decline of more than 69% below the pre-crisis peak. Recovery of credit was slow, with new credit still at 20% below the pre-crisis peak in 2019. The right-hand figure plots the supply of credit that we insert into the model. We target the initial decline of credit by 69% in the data and maintain that decline for two years. For subsequent years, we do not impose the actual path of credit, as persistent financial tightness and changes in subsidies around the Euro crisis are likely to drive the depressed supply of credit. Rather, we take the conservative approach of allowing the financial shock to wear off as an AR(1) process; with $\kappa_t = (\kappa_{t-1} - \kappa)\xi + \kappa$, where we choose $\xi = 0.66$ per quarter in line with the degree of persistence of financial shocks in Gertler and Karadi (2011). This means that our estimates on the contribution of tight credit to the slowdown of green patenting is a lower bound on finance's true contribution.

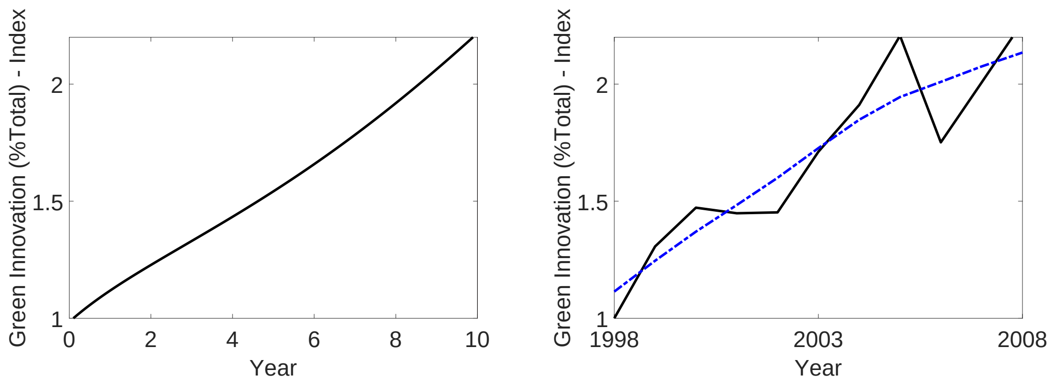
Figure 11 plots the resultant path for the fraction of goods that is produced by young firms. The blue-circled lines plot the path when the economy is hit by a financial crisis in year 10 which raises κ for two years, after which it smoothly returns to its initial steady-state value (of zero) within three years. The black-solid line is the initial transition path. The left-hand figure plots the full transition, while the right-hand figure zooms in on the 15 years around the financial crisis. The figure shows that the green transition

Figure 8: Transition - Fraction of Goods Produced using Green Technologies



Notes: The figure plots the path of the fraction of products that is produced with green technologies (F in the model).

Figure 9: Target - Path of Green Innovation (% of Total) over Transition



(a) Model (b) Data

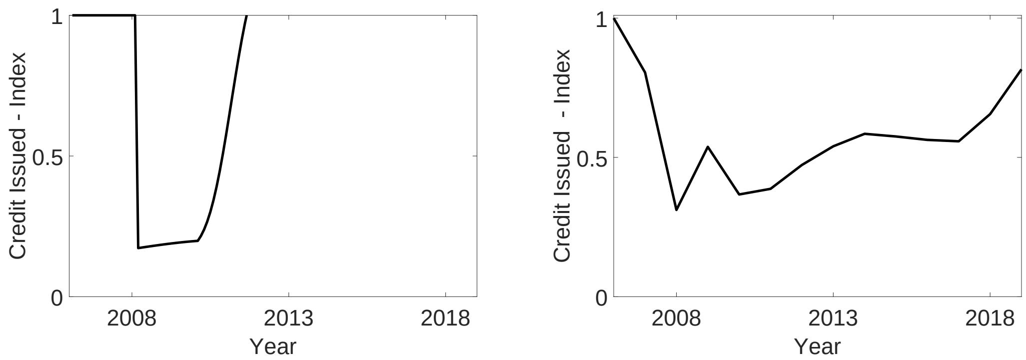
Notes: The figure plots the fraction of innovations by entrants and incumbents that is in the green technology. The left-hand figure plots the path of green patenting in the model, the right-hand figure plots the path of green patenting in the data. Both figures are indexed to 1 in the first year. The data figure is weighted by citations to adjust for differences in quality across patents.

is temporarily but significantly slowed by the tightening of financial conditions. As noted before, there is no notion of "green finance" in our model: both green innovation and dirty innovation are equally subject to credit frictions. Any difference between green and dirty innovation's exposure to finance comes from differences in the age distribution of firms that engage in (respectively) dirty or green innovation. More specifically, the slowdown of the transition in Figure 11 is driven by the crisis' disproportionate effect on younger firms.

This mechanism is further illustrated by Figure 12, which shows that the model matches the fact, established in Section 2, that the financial shock caused a large decline in green patents by younger firms, despite the fact that there has only been a single financial shock that does not differentially affect green and dirty innovation. The financial crisis affected green patents disproportionately for younger firms and we know that these are the firms which play a leading role in green innovation particularly in the early stage of the transition process.

The decline in green innovation during the financial shock in turn has long-term effects on the stock of pollution. As noted by Acemoglu et al. (2016), CO2 emissions have slow depreciation rates and a temporary slowdown of the green transition can therefore have persistent effects on the climate. To show the magnitude of the deficit in pollution reduction, Figure 13(a) plots the deviation of the fraction of all products that is produced with green technologies from the path without a financial crisis. Because emissions are proportional to that fraction, the figure plots how far emission reductions are behind schedule. At its peak, green production is 10% behind the no-crisis path. Figure 13(b) shows how the slowdown in green transition induced by the financial shock, in turn affects cumulative additional emissions. Around 10 years after the financial crisis, cumulative additional emissions peak at 45% of pre-crisis annual emissions. After that there is a slight catch up, because the low fraction of green products in the economy imply a higher average markup (and thus innovation value) of green production than in the no-crisis

Figure 10: Target - Path of Credit during the Financial Crisis



(a) Model (b) Data

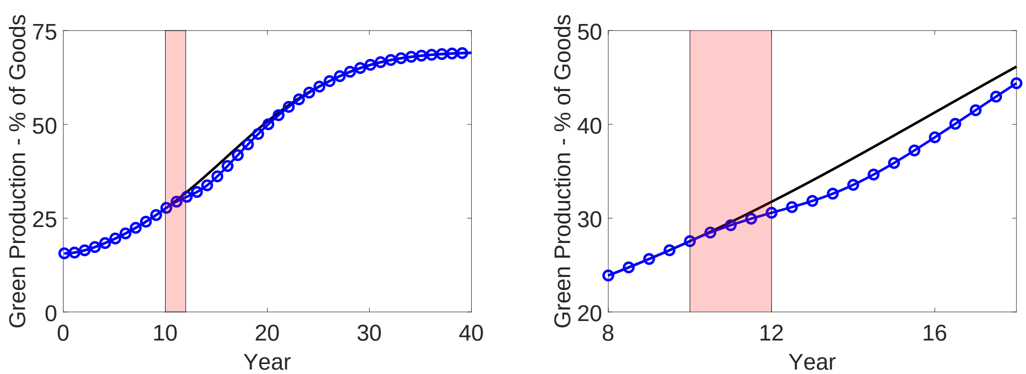
Notes: The figure plots the path of new issued credit in the model (left) and the data (right), indexed to 1. Data is based on issuance of bonds and syndicated loans. Horizontal axes for the theoretical graph is years since the start of the transition.

path. The cumulative additional emissions converge to just over 40%, which means that the financial crisis caused the emissions stock to increase by around half a year of pre-crisis emissions.

7 Conclusion

This paper proposes that green innovation is disproportionately affected by financial disruptions when the economy is in transition towards a low-emission steady state. We hypothesize that younger firms have a comparative advantage in green innovation at the early stages of a transition, as they have not ac-

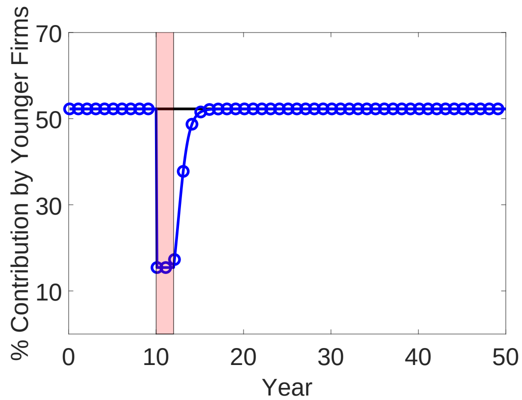
Figure 11: Transition - Fraction of Goods Produced using Green Technologies with Financial Crisis



(a) Full Transition (b) Financial Crisis

Notes: The figure plots the path of the fraction of products that is produced with green technologies (F in the model). A financial crisis hits the economy in year 10 and lasts for 2 years (the red-shaded area), after which financial costs reduce to steady state levels with an AR(1) persistence of 0.66 per quarter. Black-solid lines plot the path of F in absence of a financial crisis, blue-circled lines plot the new path. The horizontal axis counters the number of years since the start of the transition.

Figure 12: Transition - Fraction of Green R&D by Younger Firms



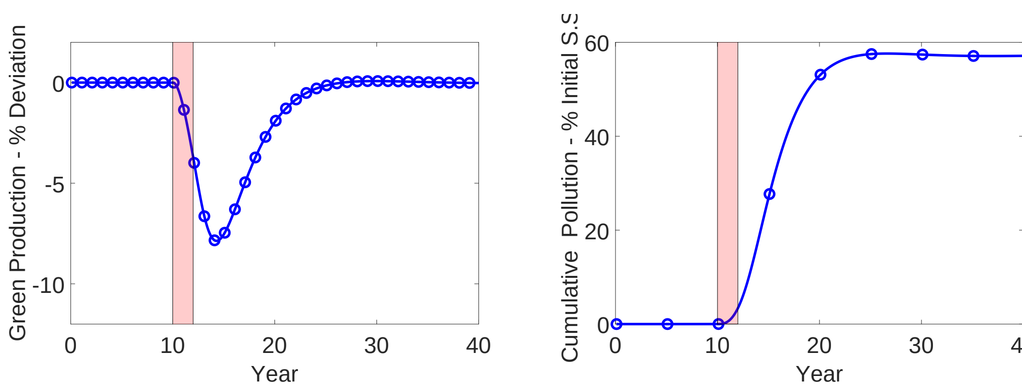
Notes: The figure plots the fraction of green R&D for which young firms are responsible. A financial crisis hits the economy in year 10 and lasts for 2 years (the red-shaded area), after which financial costs reduce to steady state levels with an AR(1) persistence of 0.66 per quarter. The horizontal axis counters the number of years since the start of the transition.

cumulated expertise in dirty innovation that older firms have. As young firms are financially constrained, a tightening of overall credit will thus endogenously affect green innovation more than dirty innovation.

We develop this argument using an innovation-led growth model with firm dynamics where: (i) firms expand their portfolio of products by engaging in either dirty or green innovation in order to improve existing product lines; (ii) the choice between green or dirty innovation is path dependent, in the sense that it becomes increasingly less costly for a firm to innovate in green or dirty innovation, respectively, if they already produce a large number of green or dirty products; (iii) firms are subject to credit constraints that depend on their maturity, with less mature firms facing the tighter constraints.

We combine these elements in order to match three empirical facts in the data around the Global Financial Crisis. First, we find that the rise in the share of green patents in total patents stagnates after

Figure 13: Transition - Deviation of Emissions from No-Crisis Path



(a) Deviation of Emissions (b) Cumulative Additional Emissions

Notes: The figure plots the path of the fraction of products that is produced with green technologies (F in the model). A financial crisis hits the economy in year 10 and lasts for 2 years (the red-shaded area), after which financial costs reduce to steady state levels with an AR(1) persistence of 0.66 per quarter. Black-solid lines plot the initial path of the variables, blue-circled lines plot the new path. The horizontal axis counters the number of years since the start of the transition.

the Global Financial Crisis. Second, we show that firm-level exposure to the Global Financial Crisis has a significantly negative effect on subsequent green patenting but not dirty patenting. Third, we show that below-median age firms were responsible for a large share of green innovation prior to the Global Financial Crisis, and that the negative effect of the crisis on green patenting is driven by them.

A key novelty of our analysis is that a financial crisis has a state-dependent effect on green innovation. As there is no intrinsic difference between financing costs of green and dirty innovations, both would be equally affected in the steady state. This changes when the economy is in a transition from a steady state in which most products are produced with dirty, polluting technologies, to one where most products are produced with clean technologies. The driver of the transition is policy: an increase in the relative profitability of producing goods with a clean rather than a dirty technology, for instance through the introduction of a carbon tax. We structurally estimated the model's initial high pollution steady state to match the patterns of innovation between 1985 and 1995 and then implement a pollution tax that is sufficiently large to reduce emissions by 65%. As the vast majority of patents in the initial steady state is in the dirty technology, incumbents initially have limited incentives alter the direction of innovation towards green innovation. As a result, the model matches the empirical fact that young firms invest in green innovation at comparatively high rates before the Global Financial Crisis, and that most green innovations were produced by young firms.

We then simulate a financial crisis that occurs early in the green transition. We found that the decline in credit induced by the financial crisis is able to explain a significant share - around 60% - of the slowdown in green patenting after 2008. Moreover, the financial crisis induces a persistent increase in emissions, causing the stock of global emissions to be the equivalent of more than half a year of pre-transition emissions higher as a result of the crisis.

Our analysis has policy implications, in particular for monetary policy and its interplay with the green transition. One implication of our analysis, is that excessively high increases in Central Bank interest rates or other similar devices aimed at reducing inflation to restore price stability, may slow down the transition towards a green economy, which in turn may defeat the declared purpose of restoring price stability in the long run.

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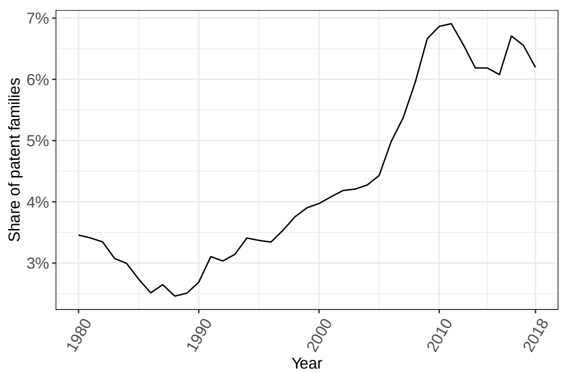
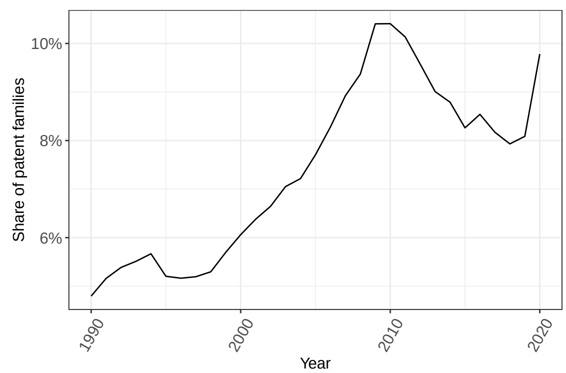
Online Appendix for 'Lost in Transition'

A Additional Figures and Tables

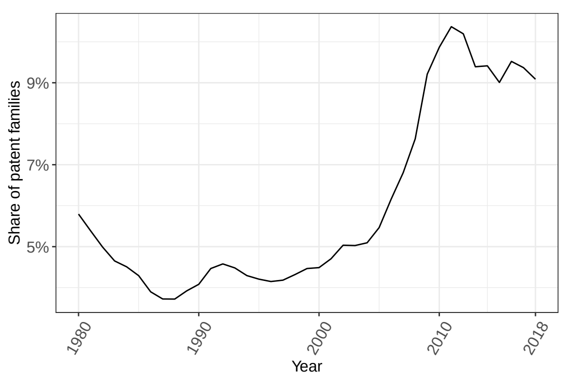
Figure A1: Alternative Trends in Green Patents as a Percentage of Total Patents



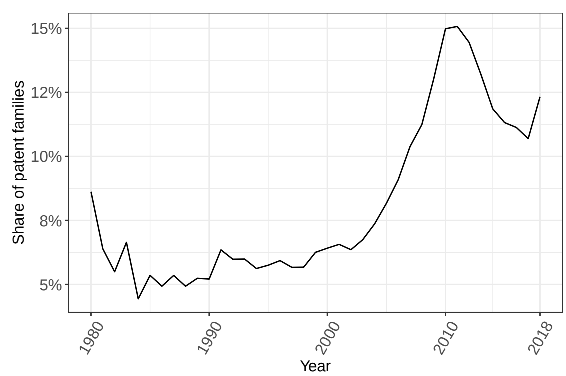
(a) Baseline



(b) Citation-Weighted



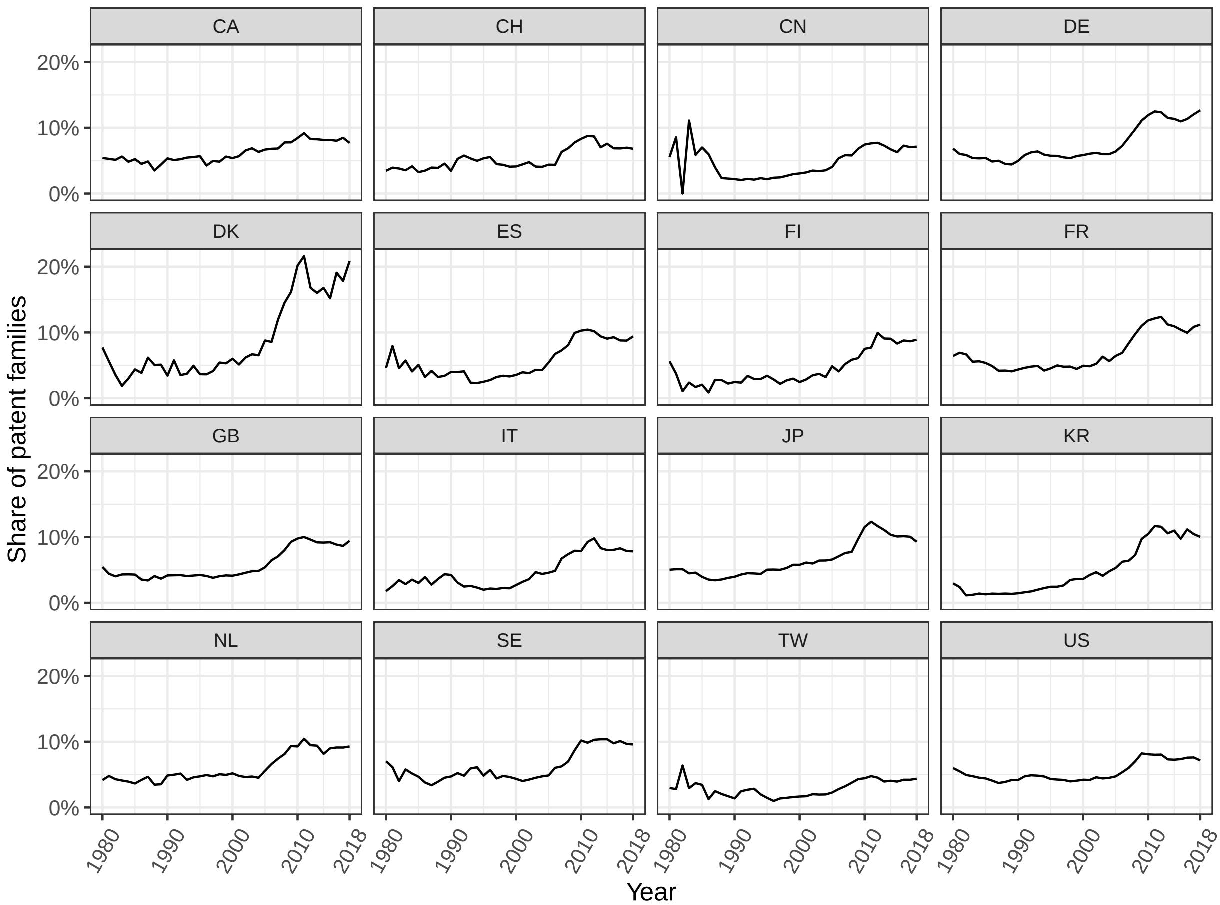
(c) OECD



(d) Triadic

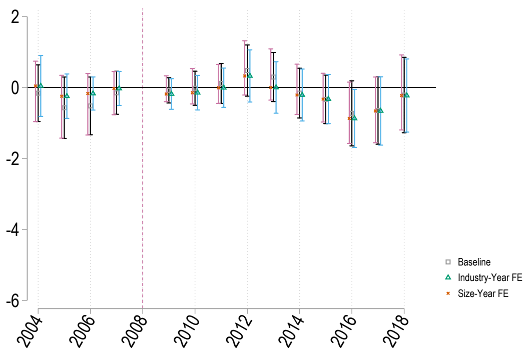
Notes: Source: % patents in environmental management, water-related adaption technologies, biodiversity and ecosystem health, climate-change mitigation (OECD classification - Hascic and Migotto 2015) - PATSTAT data. Triadic patents are patents filed at each of the European Patent Office, the U.S. Patent Office and the Japan Patent Office.

Figure A2: Global Trend: Green Patents as a Percentage of Total Patents

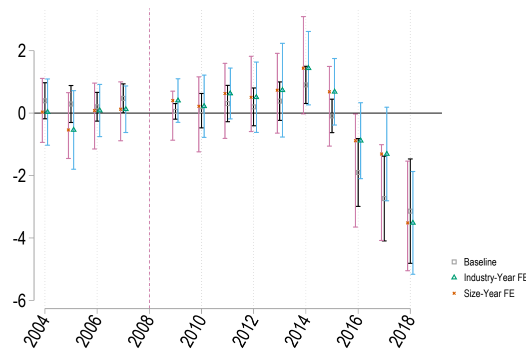


Notes: Source: % patents in environmental management, water-related adaption technologies, biodiversity and ecosystem health, climate-change mitigation (OECD classification - Hascic and Migotto 2015) - PATSTAT data.
CA: Canada, CH: Switzerland, CN: China, DE: Germany, DK: Denmark, ES: Spain, FI: Finland, FR: France, GB: United Kingdom, IT: Italy, JP: Japan, KR: Republic of Korea, NL: Netherlands, SE: Sweden, TW: Taiwan, US: United States.

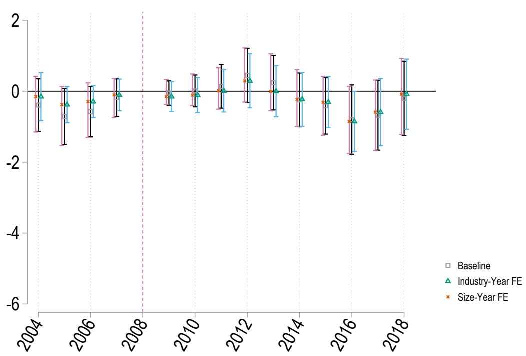
Figure A3: Robustness: Effect of Global Financial Crisis on Patent Applications



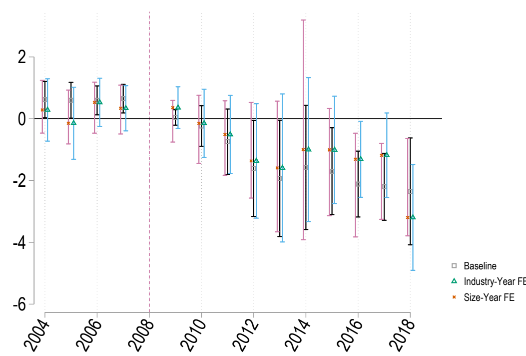
(a) Non-Green Patents



(b) Green Patents



(c) Non-Green (Stock Controls)



(d) Green (Stock Controls)

Notes: The figure plots the effect of exposure to Commerzbank on non-green patents (left) and green patents (right). Patents are classified as green if at least one IPC code relates to environmental management, water-related adaption technologies, biodiversity and ecosystem health, climate-change mitigation (OECD classification - Haščič and Migotto 2015). Confidence bounds are at the 95% level based on firm-clustered standard errors.

B Proofs and Derivations

B.1 Proof of Proposition 1

The Bellman equation reads

$$rV_t(J,k) - \dot{V}_t(J,k) = \max_{x^c,x^d} \left\{ \begin{array}{c} \sum_{j\in J^A} \pi^{h_j \tilde{h}_j} Y_t \\ +\sum_{l\in J^U} \zeta \mathbb{E}\left[V_t(J\cup J^A\{h_l h_l'\},k)\right. \\ +\nu\left[V_t(J,k+1) - V_t(J,k)\right]\cdot \mathbb{1}_{\{k<\overline{k}\}} \\ +\tau n^c \mathbb{E}\left[V_t(J\setminus\{ch'\},k) - V_t(J,k)\right] \\ +\tau n^d \mathbb{E}\left[V_t(J\setminus\{dh'\},k) - V_t(J,k)\right] \\ +x^c n^c\left[V_t(J\cup J^U\{c\},k)\alpha + V_t(J\cup J^U\{d\},k)(1-\alpha) - V_t(J,k)\right] \\ +x^d n^d\left[V_t(J\cup J^U\{d\},k)\alpha + V_t(J\cup J^U\{c\},k)(1-\alpha) - V_t(J,k)\right] \\ -w_t\tilde{r}(k)\left[(\phi^c)^{-1}(x^c)^{\psi} n^c + (\phi^d)^{-1}(x^d)^{\psi} n^d\right] \end{array} \right\}.$$

Use $\mathbb{E}\pi^d = F\pi^{dc} + (1-F)\pi^{dd}$ and $\mathbb{E}\pi^c = F\pi^{cc} + (1-F)\pi^{cd}$. Inserting the guess:

$$\begin{aligned} \rho\left(V^c(k)n^c + V^d(k)n^d + \hat{V}^c(k)\hat{n}^c + \hat{V}^d(k)\hat{n}^d\right) = \max_{x^c,x^d} \Bigg\{ & \left(\mathbb{E}\pi^c n^c + \mathbb{E}\pi^c n^d\right) \\ & + \zeta\left[\widehat{n}^c\left(V^c(k) - \widehat{V}^c(k)\right) + \widehat{n}^d\left(V^c(k) - \widehat{V}^c(k)\right)\right] \\ & - \tau\left[n^c V^c(k) - \tau n^d V^d(k)\right] \\ & + \nu\left[n^c(V^c(k+1) - V^c(k)) + n^d(V^d(k+1) - V^d(k))\right] \\ & + \nu\left[\widehat{n}^c(\widehat{V}^c(k+1) - \widehat{V}^c(k)) + \widehat{n}^d(\widehat{V}^d(k+1) - \widehat{V}^d(k))\right] \\ & + x^c(k)n^c(\alpha\widehat{V}^c + (1-\alpha)\widehat{V}^d) + x^d(k)n^d(\alpha\widehat{V}^d + (1-\alpha)\widehat{V}^c) \\ & - \omega\tilde{r}(k)\left[(\phi^c)^{-\psi}(x^c(k))^{\psi} n^c + (\phi^d)^{-\psi}(x^d(k))^{\psi} n^d\right] \Bigg\} \end{aligned}$$

where $\omega \equiv w_t/Y_t$. If a firm is in the absolving state, such that $V^h(k) = V^h(k+1)$, the equation becomes:

$$\begin{aligned} \rho\left(V^c(k)n^c + V^d(k)n^d + \hat{V}^c(k)\hat{n}^c + \hat{V}^d(k)\hat{n}^d\right) = \max_{x^c,x^d} \Bigg\{ & \left(\mathbb{E}\pi^c n^c + \mathbb{E}\pi^c n^d\right) \\ & + \zeta\left[\widehat{n}^c\left(V^c(k) - \widehat{V}^c(k)\right) + \widehat{n}^d\left(V^c(k) - \widehat{V}^c(k)\right)\right] \\ & - \tau\left[n^c V^c(k) - \tau n^d V^d(k)\right] \\ & + x^c(k)n^c(\alpha\widehat{V}^c + (1-\alpha)\widehat{V}^d) + x^d(k)n^d(\alpha\widehat{V}^d + (1-\alpha)\widehat{V}^c) \\ & - \omega\tilde{r}(k)\left[(\phi^c)^{-\psi}(x^c(k))^{\psi} n^c + (\phi^d)^{-\psi}(x^d(k))^{\psi} n^d\right] \Bigg\} \end{aligned}$$

Hence, at the optimal innovation rates, the terms in the value function read:

$$\widehat{V}^c(\overline{k}) = \left(\frac{\zeta}{\rho+\zeta}\right) V^c(\overline{k})$$
$$\widehat{V}^d(\overline{k}) = \left(\frac{\zeta}{\rho+\zeta}\right) V^d(\overline{k})$$

$$V^c(\overline{k}) = \frac{\mathbb{E}\pi^c + x^c(\overline{k})(1-\alpha)\left(\frac{\zeta}{\rho+\zeta}\right) V^d(\overline{k}) - \omega\tilde{r}(\overline{k})(x^d(\overline{k}))^{\psi}\phi^{-1}}{\rho+\tau - x^c(\overline{k})\alpha\left(\frac{\zeta}{\rho+\zeta}\right)}$$
$$V^d(\overline{k}) = \frac{\mathbb{E}\pi^d + x^d(\overline{k})(1-\alpha)\left(\frac{\zeta}{\rho+\zeta}\right) V^c(\overline{k}) - \omega\tilde{r}(\overline{k})(x^d(\overline{k}))^{\psi}\phi^{-1}}{\rho+\tau - x^d(\overline{k})\alpha\left(\frac{\zeta}{\rho+\zeta}\right)}$$

For firms not in the absolving state, there is an additional term for changes in maturity.

$$\widehat{V}^c(k) = \left(\frac{\zeta}{\rho+\zeta+\nu}\right) V^c(k) + \left(\frac{\nu}{\rho+\zeta+\nu}\right)\widehat{V}^c(k+1)$$
$$\widehat{V}^d(k) = \left(\frac{\zeta}{\rho+\zeta+\nu}\right) V^d k) + \left(\frac{\nu}{\rho+\zeta+\nu}\right)\widehat{V}^d(k+1)$$

$$V^c(k) = \frac{\mathbb{E}\pi^c + x^c(\overline{k})(1-\alpha)\left(\frac{\zeta}{\rho+\zeta}\right) V^d(\overline{k}) - \omega\tilde{r}(\overline{k})(x^d(\overline{k}))^{\psi}\phi^{-1} + \nu V^c(k+1)}{\rho+\tau - x^c(\overline{k})\alpha\left(\frac{\zeta}{\rho+\zeta}\right)+\nu}$$
$$V^d(\overline{k}) = \frac{\mathbb{E}\pi^d + x^d(\overline{k})(1-\alpha)\left(\frac{\zeta}{\rho+\zeta}\right) V^c(\overline{k}) - \omega\tilde{r}(\overline{k})(x^d(\overline{k}))^{\psi}\phi^{-1} + \nu V^c(k+1)}{\rho+\tau - x^d(\overline{k})\alpha\left(\frac{\zeta}{\rho+\zeta}\right)+\nu}$$

B.2 Derivations of aggregate variables

To find the equilibrium wage, start from the expression for aggregate output:

$$\ln Y = \int_0^1 \ln q_j + \ln y_j dj,$$

where firm subscripts are omitted for readability. Inserting the unit-elastic demand functions gives:

$$\ln Y = \int_0^1 \ln q_j dj + \ln Y + \int_0^1 \ln p_j dj,$$

Note that the unit measure of products can be separated into the fraction F that is produced with green inputs, and the remainder $1-F$ which is produced with dirty inputs. Further note that, by the law of large numbers, a fraction F of all products is produced with a green second-best competitor, regardless of whether the first-best (producing) firm uses green or dirty technology itself:

$$\ln Y = \int_0^1 \ln q_j \, dj + \ln Y - \int_0^F \ln p_j \, dj - \int_F^1 \ln p_j \, dj,$$

$$\ln Y = \int_0^1 \ln q_j \, dj + \ln Y - F \ln mc^c - (1-F) \ln mc^d - F\left(F \ln \mu^{cc} + (1-F)\mu^{cd}\right) - (1-F)\left(F \ln \mu^{dc} + (1-F)\mu^{dd}\right)$$

Inserting $mc^c = w$ while $mc^d = w \times \tilde{p}^o$, while $\mu^{cc} = \mu^{dd} = \lambda$, $\mu^{cd} = \mu \times \tilde{p}^o$, and $\mu^{dc} = \mu / \tilde{p}^o$ gives

$$\ln w = \int_0^1 \ln q_j \, dj - F\overline{\ln \mu^c} - (1-F)\overline{\ln \mu^d},$$

$$\overline{\ln \mu^c} = F \ln \mu^{cc} + (1-F) \ln \mu^{cd} \quad \text{and} \quad \overline{\ln \mu^d} = F \ln \mu^{cd} + (1-F) \ln \mu^{cc} + \ln \tilde{p}^o.$$

Taking exponents yields the standard result that the wage is equal to productivity, marked down by the effects of producer market power and the relative cost of dirty production:

$$w = \exp\left(\int_0^1 \ln q_j \, dj\right)\left(\exp\left(\overline{\ln \mu^c}\right)^F \exp\left(\overline{\ln \mu^d}\right)^{(1-F)}\right)^{-1}.$$

Next we derive an expression for aggregate output from the resource constraint. First, define L^p as the total inputs required for production:

$$\begin{aligned} L^p &= L^c + \tilde{L}^d \\ &= \int_0^F l_j^c \, dj + \int_F^1 \tilde{p}^o l_{ij}^d \, di \\ &= \int_0^F y_j^c \, dj + \int_F^1 y_j^d \, di \end{aligned}$$

This can be written in terms of output prices by inserting the unit-elastic demand function:

$$L^p = Y\left(\int_0^F \left(p_j^c\right)^{-1} dj + \int_F^1 \left(p_j^d\right)^{-1} dj\right)$$

Rewriting prices in terms of markups and marginal costs gives:

$$L^p = \frac{Y}{w}\left(\int_0^F (\mu_j)^{-1} \, dj + (\tilde{p}^o)^{-1} \int_F^1 (\mu_j)^{-1} \, dj\right)$$

This gives the required expression for output:

$$Y = L^p \times \exp\left(\int_0^1 \ln q_j \, dj\right) \times \mathscr{M}$$

where

$$(\overline{\mu^c})^{-1} = F(\mu^{cc})^{-1} + (1-F)(\mu^{cd})^{-1} \quad \text{and} \quad (\overline{\mu^d})^{-1} = \left(F(\mu^{cd})^{-1} + (1-F)(\mu^{cc})^{-1}\right)\frac{1}{\tilde{p}^o},$$

$$\mathcal{M} = \frac{\exp(-F \times \overline{\ln \mu^c} - (1-F) \times \overline{\ln \mu^d})}{F(\overline{\mu^c})^{-1} + (\tilde{p}^o)^{-1}(1-F)(\overline{\mu^d})^{-1}}.$$

The first term is production input, the second term is the CES productivity index, and the third term measures the productivity loss due to the misallocation associated with markup and production-cost heterogeneity. A term similar to the latter is derived in Peters (2020).

C Computation

The balanced growth path equilibrium is found by solving the system of detrended equilibrium equations as a fixed point. The algorithm works as follows:

1. Solve the fixed point:

   (a) Guess a firm-size and firm-age distribution, calculate innovation rates, creative destruction rates, and aggregate variables.

   (b) Given these calculations, obtain a new firm-size and firm-age distribution using the law of motion: Model with lags (claim: can use old equilibrium firm-size distribution)

$$\begin{aligned}
\dot{M}_{n^c=\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}} &= \tau\left(M_{n^c=\overline{n}^c+1, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}}\right)(\overline{n}^c+1) \\
&+ \tau\left(M_{n^c=\overline{n}^c, n^d=\overline{n}^d+1, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}}\right)(\overline{n}^d+1) \\
&+ \zeta\left(M_{n^c=\overline{n}^c-1, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c+1, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}}\right)(\overline{\widehat{n}}^c+1) \\
&+ \zeta\left(M_{n^c=\overline{n}^c, n^d=\overline{n}^d-1, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d+1, k=\overline{k}}\right)(\overline{\widehat{n}}^d+1) \\
&+ x^c(\overline{k})\left(M_{n^c\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c-1, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}}\right)\overline{n}^c\alpha \\
&+ x^d(\overline{k})\left(M_{n^c\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d-1, k=\overline{k}}\right)\overline{n}^d\alpha \\
&+ x^c(\overline{k})\left(M_{n^c\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d-1, k=\overline{k}}\right)\overline{n}^c(1-\alpha) \\
&+ x^d(\overline{k})\left(M_{n^c\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c-1, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}}\right)\overline{n}^d(1-\alpha) \\
&+ \nu\left(M_{n^c=\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}-1}\right) \\
&- \nu\left(M_{n^c=\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}}\right) \\
&- \left(x^c(k)+\tau\right)\left(M_{n^c=\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}}\right)\overline{n}^c \\
&- \left(x^d(k)+\tau\right)\left(M_{n^c=\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}}\right)\overline{n}^d \\
&- \zeta\left(M_{n^c=\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}}\right)\overline{\widehat{n}}^c \\
&- \zeta\left(M_{n^c=\overline{n}^c, n^d=\overline{n}^d, \widehat{n}^c=\overline{\widehat{n}}^c, \widehat{n}^d=\overline{\widehat{n}}^d, k=\overline{k}}\right)\overline{\widehat{n}}^d
\end{aligned}$$

Where the steady state has constant measure of firms of all states:

$$\dot{M}_{n^c=\overline{n}^c, n^d=\overline{n}^d, a=\overline{a}} = 0 \qquad \forall \quad \overline{n}^c, \overline{n}^d, \overline{a}$$

The solutions from defining the transition matrix T that has $(n^{max})^4 \cdot \|\overline{k}\| \times (n^{max})^4 \cdot \|\overline{k}\|$

$$\dot{M} = T \times M + E \quad \Rightarrow \quad M = -(T)^{-1} E$$

(c) Iterate until the innovation rates, creative destruction rates, and aggregate variables converge.

2. In the structural estimation, the resulting moments are then compared to the targets using the penalty function described in the main text. The parameters are updated along either a genetic algorithm or particle swarm algorithm – to optimize fit – until the penalty function is minimized.

The transitional dynamics are numerically solved using the following algorithm:

1. Create a fine grid with a T-year horizon, allowing each year to consist of $\tilde{T}$ instances.
2. Guess an initial value function of innovation activities $V()$ equal to the new steady-state level for each technology h and each maturity type k at each point of the grid. Similarly guess the paths of wages w/Q and output Y/Q at their new steady-state level.
3. Initialize the firm-size and technology distributions M to their original steady state.
4. Iterate over the path of the value function as follows:
   (a) Solve the static optimization problem and the dynamic innovation decisions for incumbents and entrants for each point on the grid using the initial guess for $V()$.
   (b) Given the innovation and static decisions, simulate the development for a large (N) number of products and track the innovation step-sizes λ in $N \times (T\tilde{T})$ matrix Λ and similarly a matrix of ownership types using a forward loop over the grid.[12]
   (c) Update the value function using the new sequences for Y, w, the firm-type and -size distribution, and distributions for markups and λs implied by Λ. This involves calculating:
      i. the expectation of profits $\pi_{kt}(\phi_i, \lambda_{ij})$ at each instance t on the grid $t = 1, .., T$ separately for each cohort of patents k.
      ii. This is used to calculate the value of producing an additional product at time kt.
   (d) Use the resulting value for each type on each point of the grid as the guess for $V()$ in step (a) in the next iteration. Continue until the path of the value function converges.

[12] This simulation is needed because the changing composition of firm types means the distribution of realized λs has no analytical representation. I then use the resulting distribution of markups to calculate the efficiency wedge as in (24), as well as a path for Y and w. These serve as the basis for the algorithm's next iteration.